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# Outsourcing through Competition: What is the Best Competition Parameter?

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## Abstract

In this paper we consider a single buyer who wants to outsource the manufacturing of a product to  $N$  potential suppliers. The buyer's objective is to maximize the service level she receives from the suppliers. The suppliers compete for the buyer's demand based on a competition parameter which the buyer announces along with an allocation rule. We model each supplier as a make-to-stock queueing system. Using a simple proportional allocation function, we compare two competition parameters: service level and inventory level. We show that inventory competition creates a higher overall service level for the buyer. We also show an optimal form of competition parameter which can induce the maximum feasible service level. Our base model shows the results for the competition between identical suppliers. We then extend the results to a case where the suppliers are heterogeneous.

Keywords: Outsourcing, Inventory Competition, Service Competition, Optimal Mechanism

## 1. Introduction

Companies are increasingly using criteria other than the product price to evaluate the performance of their suppliers. Royal Philips Electronics, for example, has a supplier rating system in which cost weighs only 15% among other service and innovation related criteria<sup>1</sup>. As another example, Hawker Beechcraft, a manufacturer of business jets, has a supplier rating system in which price/cost weighs only 10% among other service and quality related criteria<sup>2</sup>. Because of the availability of multiple suppliers and the existence of powerful buyers, who can set the procurement price, many suppliers try to distinguish themselves through the quality of service they provide. On the other hand, suppliers' quality of service can play an important role in the performance of the buyer. For example, the suppliers' reliability to deliver on time (for make-to-order suppliers) or the availability of the product when the buyer needs it (for make-to-stock suppliers) can have a significant impact on the buyer's operations and her ability to fulfill the demand she faces.

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<sup>1</sup> [http://www.philips.com/shared/assets/company\\_profile/downloads/Supplier\\_Rating.pdf](http://www.philips.com/shared/assets/company_profile/downloads/Supplier_Rating.pdf)

<sup>2</sup> [http://www.hawkerbeechcraft.com/supply\\_chain/files/srs\\_supplier\\_training.pdf](http://www.hawkerbeechcraft.com/supply_chain/files/srs_supplier_training.pdf)

One of the mechanisms that buyers use to induce high service level in their supply base is competition. Instead of negotiating for high service level with each individual supplier, the buyer can specify a criterion (a performance measure) and an allocation rule based on which the suppliers compete for the buyer's demand. We call this criterion the *competition parameter*. The competition parameter could be a service measure (e.g. fill rate, probability of on-time delivery, etc.), or other performance measures that determine the service level provided by suppliers (e.g. production capacity, inventory level, etc.). The allocation rule is such that each supplier's share of demand is increasing in the level of competition parameter provided by the supplier. To maximize his own profit, each supplier chooses an optimal level of the competition parameter. The buyer then allocates the demand based on the allocation rule and the guaranteed levels of competition parameter. In general, competition can improve the overall service level while saves the hassle of negotiation processes.

Firms' competition for demand share has been studied extensively in the literature. We can categorize these papers according to the parameter based on which the firms compete. Those who study competition based on inventory level include Karjalainen (1992), Lippman & McCardle (1997), Parlar (1988), Wang & Gerchak (2001), Mahajan & Van Ryzin (2001a, 2001b), Cachon (2003), Netessine & Rudi (2003), and Li & Ha (2008). There are others who study firms' competition based on service level. Among these papers one can refer to Li & Lee (1994), Lederer & Li (1997), Hall & Porteus (2000), So (2000), Armony & Haviv (2003), Bernstein & Federgruen (2004), Boyaci & Gallego (2004), Allon & Federgruen (2007 & 2008), Xiao and Yang (2008), and Wu (2012).

All the above-mentioned papers, however, consider the competition between firms in a market with several customers. In these papers, firms compete to attract a larger proportion of customers. There is another stream of research that (like this paper) focuses on the competition between suppliers who compete for the demand share of a single buyer. Again, we can categorize these papers according to the parameter based on which the suppliers compete. Gilbert & Weng (1998), Ha et al (2003), and Jin & Ryan (2012), Benjaafar et al (2007) and Elahi et al (2012) all study the competition between suppliers when the share of demand allocated to each supplier depends on the service level the supplier guarantees.

Gilbert & Weng (1998) model a principal who allocates demand to two competing agents (service facilities). The identical agents decide about their costly service rates to attract more demand shares. The principal either allocates the demand to the agents from a single queue or from separate queues (equal expected waiting times). They show the conditions which one allocation might be superior to the other

one. Ha et al (2003) model two suppliers who compete for supply to a customer with deterministic demand. When the identical suppliers compete based on delivery frequency, the authors (using an EOQ model) show an allocation scheme which minimizes the customer's inventory cost. Jin & Ryan (2012) model two identical make-to-stock suppliers who compete based on both price and service level (fill rate) for demand shares of a single buyer. The buyer uses an allocation function in which the allocated demand is proportional to an exponential function. This allocation function is characterized by a parameter which shows the relative importance of price versus service level. The authors show the optimal value of this parameter which minimizes the buyer's cost.

Benjaafar et al (2007) compare two competition mechanisms: supplier allocation (SA) and supplier selection (SS). In a supplier allocation (SA) mechanism, each supplier receives a share of the buyer's demand which increases with the service level that supplier provides. In a supplier selection (SS) mechanism, the buyer selects only one supplier to receive the entire demand. The probability of a supplier being selected increases by the service level he provides. They show (SS) can result in higher service levels. In addition to service level, Benjaafar et al (2007) introduce another competition parameter. The authors show a reformulation of their problem in which they choose the demand-independent component of the service cost (which they name it supplier's *effort*) as the competition parameter. They show that when the demand is allocated proportional to a power function of this competition parameter, supplier service level can be maximized. Benjaafar et al (2007) acknowledge that the service-based and effort-based competitions can lead to different equilibrium service levels. However, they do not actually compare the two types of competition. In this research, we focus on the simplest form of allocation function (simple proportional) and compare the equilibrium service levels under different competition parameters. Elahi et al (2012) show an optimal form of allocation function for a service-based competition which can result in maximum buyer's profit. A review of service-based outsourcing can be found in Zhou & Ren (2010).

There are other research papers that study the suppliers' competition based on non-service measures. Bell & Stidham (1993) model the competition between two servers in a marketplace when the demand share of each server depends on the server's capacity level. Cachon & Zhang (2007) model the competition between two identical make-to-order suppliers who supply to a single buyer. The buyer allocates the demand to the suppliers based on their capacity levels. The authors show the impact of different allocation schemes. In their model, the buyer's objective is to maximize the service level

provided by the suppliers. They show the form of a linear allocation function which can produce the best results. The authors also mention the possibility of allocating demand based on service level (defined in terms of waiting time). They show that the maximum feasible capacity that can be induced in a capacity competition could, under certain conditions, be higher than the maximum feasible capacity that can be induced in a service competition. Although the general message of this result is aligned with what we show in this paper, their result is different from ours since they focus only on the maximum feasible capacity, not the equilibrium point of a competition based on a given allocation function.

In all the above-mentioned papers, the focus is on the allocation rule based on which the demand is allocated (for a given competition parameter). That is, they mostly try to address the question of what form of allocation rule can provide higher service levels. We can conclude from this stream of research that the form of the allocation rule has a significant impact on the intensity of the competition. Moreover, by using rather complicated allocation rules, the buyer can induce the maximum feasible service level.

To the best of our knowledge, none of the existing papers in this field focus on the impact of competition parameter. Although both Cachon & Zhang (2007) and Benjaafar et al (2007) acknowledge that different competition parameters can lead to different competition intensity, none of them actually compare the equilibrium points of competitions based on different competition parameters. In this research, we want to fill this gap by comparing the average service level that the buyer can achieve under different competition parameters. More specifically, we want to answer these questions: (a) does the choice of competition parameter have a significant impact on the overall service level that the buyer receives? (b) which competition parameters create higher competition intensity and therefore better results for the buyer? (c) can we find (or design) an optimal competition parameter that induces the maximum feasible service level? Answering these questions not only creates a better understanding of competition as an outsourcing mechanism, but also provides the potential opportunity to orchestrate a more effective and more practical form of competition. When the buyer uses a simple proportional allocation function along with a proper competition parameter to create high competition intensity, the simple and intuitive form of allocation function makes it easier for the buyer to communicate the competition setup to the suppliers (compared to a setup in which the buyer uses a complicated form of allocation function).

To answer the above-mentioned questions we use a stylized queueing model in which a buyer outsources to  $N$  potential make-to-stock suppliers. We first compare the competition based on the service

level with the competition based on inventory level. That is, we examine the impact of using service level (fill rate) versus inventory level (base stock level) as the competition parameter. Although the ultimate goal is to maximize the overall service level, we will show the unexpected result that the competition based on inventory provides higher service level than the competition based on the service level itself. Our base model shows the results for a case where the suppliers are identical. We then extend the results to heterogeneous suppliers. To focus on the impact of competition parameter, we use a simple proportional allocation function. We show that the buyer can design an optimal competition parameter which is capable of inducing the maximum feasible service level; even when we use a simple proportional allocation function.

The rest of this paper is organized as follows. Section 2 formulates our outsourcing problem. Section 3 solves this problem when the buyer owns the suppliers (or has the power to dictate the contract terms). The solution to this problem provides a first best solution for the buyer, which can be used as a benchmark for the solution to our competition problems. Section 4 shows the Nash equilibrium for the service and inventory competitions and compares the results of the two competitions. Section 5, shows how the supplier heterogeneity can impact the results of section 4. In section 6, we show an optimal form of competition parameter which can induce the maximum feasible service level. Concluding remarks are presented in Section 7. Appendix A provides the proofs to the theorems. Appendix B shows the results for the competition based on the number of backorders. Appendix C provides a list of all the variables and parameters used to model the problem and their definitions.

## 2. Model Formulation

We consider a supply chain in which a buyer wants to outsource her demand for the manufacturing of a product to  $N$  suppliers. Suppliers manufacture this product in a make-to-stock fashion using base-stock inventory policy. Demand at the buyer occurs according to a Poisson process with rate  $\lambda$ . The fraction of demand allocated to supplier  $i$  is denoted by  $\delta_i$ , where  $0 < \delta_i < 1$  and  $\sum_{i=1}^N \delta_i = 1$ . The parameter  $\delta_i$  can be viewed as the probability that incoming demand is allocated to supplier  $i$ <sup>3</sup>. Therefore, the arrival process at each supplier is also Poisson with a rate of  $\delta_i \lambda$ . Production times at each supplier are

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<sup>3</sup> Although a truly probabilistic allocation is unlikely in practice, it is useful in approximating the behavior of a central dispatcher that attempts to adhere to a specified market share for each supplier. It is also useful in modeling settings where demand arises from a sufficiently large number of sources. The variable  $\delta_i$  corresponds in that case to the fraction of demand sources (e.g., geographical locations) that are always satisfied from supplier  $i$ .

exponentially distributed with rate  $\mu_i$ . The assumptions of Poisson arrival process and exponential processing time (in addition to being plausible in many practical setups) make our derivations mathematically tractable. Hence, the production system at each supplier behaves like an  $M/M/1$  queue. In response to the demand share allocation, the suppliers adjust their capacity (production rate) to maintain a fixed target utilization,  $\rho_i = \delta_i \lambda / \mu_i$ ,  $0 < \rho_i < 1$ . In practice suppliers' utilization might change with a change in their allocated demand since they might not adjust their capacity exactly proportional to the change in the demand share. However, we use this assumption to make our model mathematically tractable. This is consistent with the assumptions in Benjaafar et al (2007).

Finished goods at the suppliers are managed according to a base-stock policy with base stock level  $z_i$  ( $z_i > 0$ ) at supplier  $i$ . This means that the arrival of demand at supplier  $i$  always triggers a replenishment order with the supplier's production system. Suppliers incur the inventory holding cost. That is, each supplier  $i$  incurs a holding cost  $h_i$  per unit of inventory per unit time. Moreover, each supplier incurs a production cost  $c_i$  per unit produced, and a capacity cost  $k_i$  per unit of capacity (measured in terms of the associated production rate). Each supplier's revenue is the procurement price  $p$  which he receives from the buyer per unit demand allocated to the supplier. We assume this price is set by market mechanisms, so it is the same across all suppliers.

The demand which cannot be fulfilled from on-hand inventory is backordered. That is, when a supplier is out of stock, the buyer waits until the supplier produces the backordered units. When stock-out happens at a supplier, we exclude the possibility of the buyer switching to another supplier since it can violate the demand allocation scheme. We also exclude the possibility of the buyer procuring the product from a supplier outside of the pool of competing suppliers, assuming that the product is not readily available in the market. The assumption of backordering the demand when it cannot be satisfied from on-hand inventory is consistent with the assumptions in Cachon & Zhang (2007) and Benjaafar et al (2007). Netessine et al (2006) also consider this assumption. They study the impact of customers' backordering behavior on the performance of competing firms in a market.

Backordered demand is costly for the buyer. It might have a negative impact on the buyer's production system (for instance when the buyer uses a just-in-time system), or it might delay the delivery of the product to the buyer's customers. Therefore, the buyer measures each supplier's service level in terms of fill rate,  $s_i = \Pr(I_i > 0)$ . That is, the probability that a unit demand allocated to a supplier is not backordered and can be fulfilled immediately from on-hand inventory ( $I_i$  is the inventory level at

supplier  $i$ ). Hence, the buyer's objective is to maximize the average service level she receives from her suppliers,

$$q = \sum_{i=1}^N \delta_i s_i. \quad (1)$$

Maximizing the average service level is equivalent to minimizing the expected number of backorders which in turn means minimizing buyer's backordering cost.

In order to induce higher service level in her supply base, the buyer let the suppliers compete for larger shares of her demand. We consider two types of competition: inventory competition and service competition. In service competition, each supplier is awarded a demand share based on the service level (fill rate) he guarantees. The buyer uses an allocation function  $\alpha_i^S(s_i, s_{-i})$  which specifies the fraction of demand allocated to supplier  $i$  based on his fill rate  $s_i$  and the fill rates  $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$  offered by supplier  $i$ 's competitors. In other words,  $\delta_i = \alpha_i^S(s_i, s_{-i})$ . In inventory competition, a supplier's demand share depends on his base stock level. In this type of competition, the buyer uses an allocation function  $\alpha_i^I(z_i, z_{-i})$  which specifies the fraction of demand allocated to supplier  $i$  based on his base stock level  $z_i$  and the base stock levels  $z_{-i} = (z_1, \dots, z_{i-1}, z_{i+1}, \dots, z_N)$  offered by supplier  $i$ 's competitors, which means  $\delta_i = \alpha_i^I(z_i, z_{-i})$ . Note that an increase in either  $s_i$  or  $z_i$  can increase the average service level received by the buyer. However, the allocation function based on one of the two might induce higher competition intensity and hence results in better results for the buyer.

We focus on a simple proportional form for the allocation functions. That is,

$$\alpha_i^S(s_i, s_{-i}) = \frac{s_i}{\sum_{j=1}^N s_j}, \text{ and} \quad (2)$$

$$\alpha_i^I(z_i, z_{-i}) = \frac{z_i}{\sum_{j=1}^N z_j}. \quad (3)$$

This form of allocation function is not only intuitive but also easy to communicate to the suppliers. Different forms of proportional allocation function have been widely used in the competition literature. We will show that even this simple form of allocation function can induce the maximum feasible service level when it is used with a proper competition parameter.

After the buyer announces the allocation function, suppliers simultaneously choose their base stock level<sup>4</sup>. Each supplier chooses his base stock level to maximize his expected profit; considering other suppliers' possible decisions. Then the demand will be allocated based on the allocation function. We

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<sup>4</sup> Suppliers participate in the competition as long as they can earn a non-negative expected profit.



assume that the buyer can enforce the fill rates or base stock levels chosen by the suppliers. Here, we consider a setup with complete information. That is, suppliers' cost structures are common knowledge. This is consistent with the previous papers that study demand allocation through competition. To choose the optimal base stock or service level, each supplier has to consider the trade-off between higher revenue due to larger demand shares and the higher inventory or service cost. The suppliers' expected profit under the two types of competition can then be written as

$$\pi_i^S(s_i, s_{-i}) = \alpha_i^S(s_i, s_{-i}) \lambda(p - c_i - k_i / \rho_i) - g_i^S(s_i), \quad (4)$$

$$\pi_i^I(z_i, z_{-i}) = \alpha_i^I(z_i, z_{-i}) \lambda(p - c_i - k_i / \rho_i) - g_i^I(z_i), \text{ and} \quad (5)$$

where  $g_i^S(\cdot)$  and  $g_i^I(\cdot)$  are supplier's expected holding cost in terms of fill rate and stock level, respectively. We therefore have  $g_i^I(z_i) = h_i E[I_i]$ , where  $E[I_i]$  is the expected number of on-hand inventory. Using the  $M/M/1$  queueing results, it is easy to verify that (see for example Buzacott & Shanthikumar, 1993)

$$g_i^I(z_i) = h_i \left( z_i - \frac{\rho_i}{1 - \rho_i} (1 - \rho_i^{z_i}) \right). \quad (6)$$

We also know  $s_i = \Pr(I_i > 0) = 1 - \rho_i^{z_i}$ . Therefore,

$$g_i^S(s_i) = h_i \left( \frac{\ln(1 - s_i)}{\ln \rho_i} - \frac{\rho_i}{1 - \rho_i} s_i \right). \quad (7)$$

The suppliers participate in the competition only when they can earn a non-negative expected profit. We can then rewrite the buyer's average service levels under service and inventory competitions as

$$q^S = \sum_{i=1}^N \alpha_i^S(s_i, s_{-i}) s_i \text{ and} \quad (8)$$

$$q^I = \sum_{i=1}^N \alpha_i^I(z_i, z_{-i}) (1 - \rho_i^{z_i}). \quad (9)$$

Note in the absence of competition (when the demand shares are fixed values independent of the stock or service level), there is no incentive for suppliers to hold inventory. Therefore, in the absence of competition (and other contractual commitments), suppliers hold zero inventory. That is,  $z_i = s_i = 0$ ,  $i = 1, \dots, N$ .

In sections 3 and 4, we will focus on a case with identical suppliers. That is,  $c_i = c$ ,  $k_i = k$ ,  $h_i = h$ , and  $\rho_i = \rho$ ,  $i = 1, \dots, N$ . This will allow us to provide closed form results. We will then study the impact of supplier heterogeneity through numerical examples in section 6.

### 3. Buyer's First Best Solution

In this section, we want to establish an upper bound for the average service level that the buyer can achieve. Consider a setup in which the buyer can dictate both the suppliers' base stock levels  $\mathbf{z} = (z_1, \dots, z_N)$  and the demand shares  $\boldsymbol{\delta} = (\delta_1, \dots, \delta_N)$ . This might be the case, for example, when the buyer has the market power to offer a take-it-or-leave-it contract in which the suppliers' stock levels are dictated as contract terms. Since the suppliers' decisions are directly controlled by the buyer, the average service level obtained in this problem provides an upper bound for the average service levels achievable by the buyer. Hence, the solution to this problem can serve as a benchmark for solutions obtained under any competition setup. The suppliers' expected profit can then be stated as

$$\pi_i^{FB}(z_i, \delta_i) = \delta_i \lambda (p - c - k / \rho) - g^I(z_i).$$

This profit function is decreasing concave in  $z_i$  and is equal to zero at  $z_i = z^{\max}(\delta_i)$ , where  $z^{\max}(\delta_i)$  is the unique solution to the following equation.

$$\pi_i^{FB}(z_i, \delta_i) = 0 \Rightarrow z_i - \frac{\rho}{1-\rho}(1 - \rho^{z_i}) = \frac{\delta_i \lambda (p - c - k / \rho)}{h}. \quad (10)$$

We know each supplier's service level is increasing in his base stock level. Therefore, for any demand share  $\delta_i$  allocated to supplier  $i$ , the stock level which maximizes the supplier's service level is  $z^{\max}(\delta_i)$ . Therefore, the maximum feasible service level for a supplier who receives a demand share  $\delta_i$ , will be

$$s^{\max}(\delta_i) = 1 - \rho^{z^{\max}(\delta_i)}.$$

When there is no limitation on the amount of demand that can be allocated to a supplier, it is optimal for the buyer to allocate the whole demand to just one supplier. That is,  $\delta_i = 1$ , and  $\delta_j = 0$ ,  $j \neq i$ . This means that the maximum feasible average service level is

$$\hat{q} = s^{\max}(1). \quad (11)$$

To prove this result we show that any sets of demand shares  $\boldsymbol{\delta} = (\delta_1, \dots, \delta_N)$  and any sets of service levels  $\mathbf{s} = (s_1, \dots, s_N)$  result in an average service level that cannot be greater than  $\hat{q}$ . We note that  $s^{\max}(\delta_i)$  is an increasing function, which means  $s^{\max}(\delta_i) \leq s^{\max}(1)$ . Therefore, we have

$$q = \sum_{i=1}^N \delta_i s_i \leq \sum_{i=1}^N \delta_i s^{\max}(\delta_i) \leq \sum_{i=1}^N \delta_i s^{\max}(1) = s^{\max}(1) = \hat{q}. \quad (12)$$

This means that it is best for the buyer to allocate the entire demand to just one supplier while that supplier provides the maximum feasible service level  $s^{\max}(1)$ . This is the case when there is no limitation on the amount of demand the buyer is willing to allocate to a supplier. In many situations, however, the

buyer might prefer to allocate her demand to more than one supplier to lower the risk of supply disruption. Facing suppliers with identical cost structures, one logical way to allocate the demand is to give each supplier an equally share of demand. The maximum feasible average service level under this setup will be

$$\bar{q} = \sum_{i=1}^N (1/N) s_i^{\max} (1/N) = s^{\max} (1/N). \quad (13)$$

We will show that our symmetric<sup>5</sup> allocation functions defined in (2) and (3) result in equal demand shares at the equilibrium point of the competition between identical suppliers. Therefore, we use the maximum average service level defined in (13) as a benchmark for the service levels achievable under our competition setups.

#### 4. Competition Equilibrium

The following two theorems show the equilibrium conditions of inventory and service competitions.

**Theorem 1.** *The service competition of identical suppliers with the proportional allocation function defined in (2) has a unique Nash equilibrium  $\mathbf{s}^* = (s_1^*, \dots, s_N^*)$ , where  $s_i^* = s_S^*$ ,  $i = 1, \dots, N$ , and  $s_S^*$  is the unique solution to the following equation.*

$$s_S^* = \left( \frac{N-1}{N^2} \right) \frac{\lambda(p-c-k/\rho)}{h \left[ \frac{1}{(1-s_S^*) \ln(1/\rho)} - \frac{\rho}{1-\rho} \right]} \quad (14)$$

Moreover,  $\alpha_i^S(\mathbf{s}^*) = 1/N$ ,  $i = 1, \dots, N$ . Hence, the buyer's average service level is  $q^S = s_S^*$ .

**Theorem 2.** *The inventory competition of identical suppliers with the proportional allocation function defined in (3) has a unique Nash equilibrium  $\mathbf{z}^* = (z_1^*, \dots, z_N^*)$ , where  $z_i^* = z_I^*$ ,  $i = 1, \dots, N$ , and  $z_I^*$  is the unique solution to the following equation.*

$$z_I^* = \left( \frac{N-1}{N^2} \right) \frac{\lambda(p-c-k/\rho)}{h \left[ 1 - \frac{\rho^{z_I^*+1}}{1-\rho} \ln \frac{1}{\rho} \right]} \quad (15)$$

Moreover,  $\alpha_i^I(\mathbf{z}^*) = 1/N$ ,  $i = 1, \dots, N$ . Hence, the buyer's average service level is  $q^I = 1 - \rho^{z_I^*}$ .

The subscripts  $S$  and  $I$  in  $s_S^*$  and  $z_I^*$  stand for *Service* and *Inventory* competitions, respectively. Theorem 3 compares the results of the inventory and service competitions.

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<sup>5</sup> A symmetric allocation function is an allocation function which allocates equal demand shares to two suppliers who provide the same service or stock level.

**Theorem 3.** *Using simple proportional allocation functions as stated in (2) and (3), inventory competition always results in higher average service levels,  $q^I > q^S$ .*

This means that, using simple proportional allocation function, the buyer prefers the inventory competition over the service competition. It is interesting to note that although the buyer's goal is to maximize the average service level, the competition based on inventory level can provide higher average service level than the competition based on service level itself. That is, the competition based on inventory can induce higher competition intensity. The competition intensity can be related to the shape (curvature) of the suppliers' profit functions and the service levels that maximize these functions. Using an approach similar to what we use in the proof of theorem 3, it is easy to prove that the service level that maximizes a supplier's profit under inventory competition (for any given set of other suppliers' service levels) is larger than the similar value under service competition. That is, to achieve their maximum profit, suppliers have to provide higher service levels under inventory competition (compared to service competition), which means that we have a more intense competition when inventory is our competition parameter. This observation shows that the buyer can affect the outcome of the competition by choosing the parameter based on which the suppliers compete. Competition parameters are not restricted to inventory level and service level. In section 6, we will elaborate on this point by showing the results for the competition based on expected number of backorders. Section 6 also derives the optimal form of competition parameter which can induce the maximum competition intensity.

In addition to setting the parameter based on which the suppliers compete, the buyer might have some level of control over the number of competing suppliers and their utilization level. In the remainder of this section we study the impact of these two factors. Theorem 4 characterizes the impact of the number of suppliers on the equilibrium conditions of the two competition modes.

**Theorem 4.** *The equilibrium service level  $s_s^*$  and base stock level  $z_I^*$  as stated in theorems 1 and 2 are both decreasing in  $N$  and approach zero as  $N$  approaches infinity.*

Theorem 4 suggests that the buyer prefers the minimum number of competing suppliers ( $N = 2$ ). Figure 1 shows the impact of the number of suppliers on the equilibrium service level, base stock level, and the suppliers' profit for a numerical example. In this example, we compare the equilibrium conditions of inventory and service competitions as well as the buyer's first best solution. As we expect, the buyer's first best solution provides better results for the buyer (higher service and base stock levels) while results in the lowest (zero) profit for the suppliers. Between the two competition types, we can see that the buyer

is better off under an inventory competition (higher service and base stock levels) while the suppliers are better off under a service competition (higher supplier profit).

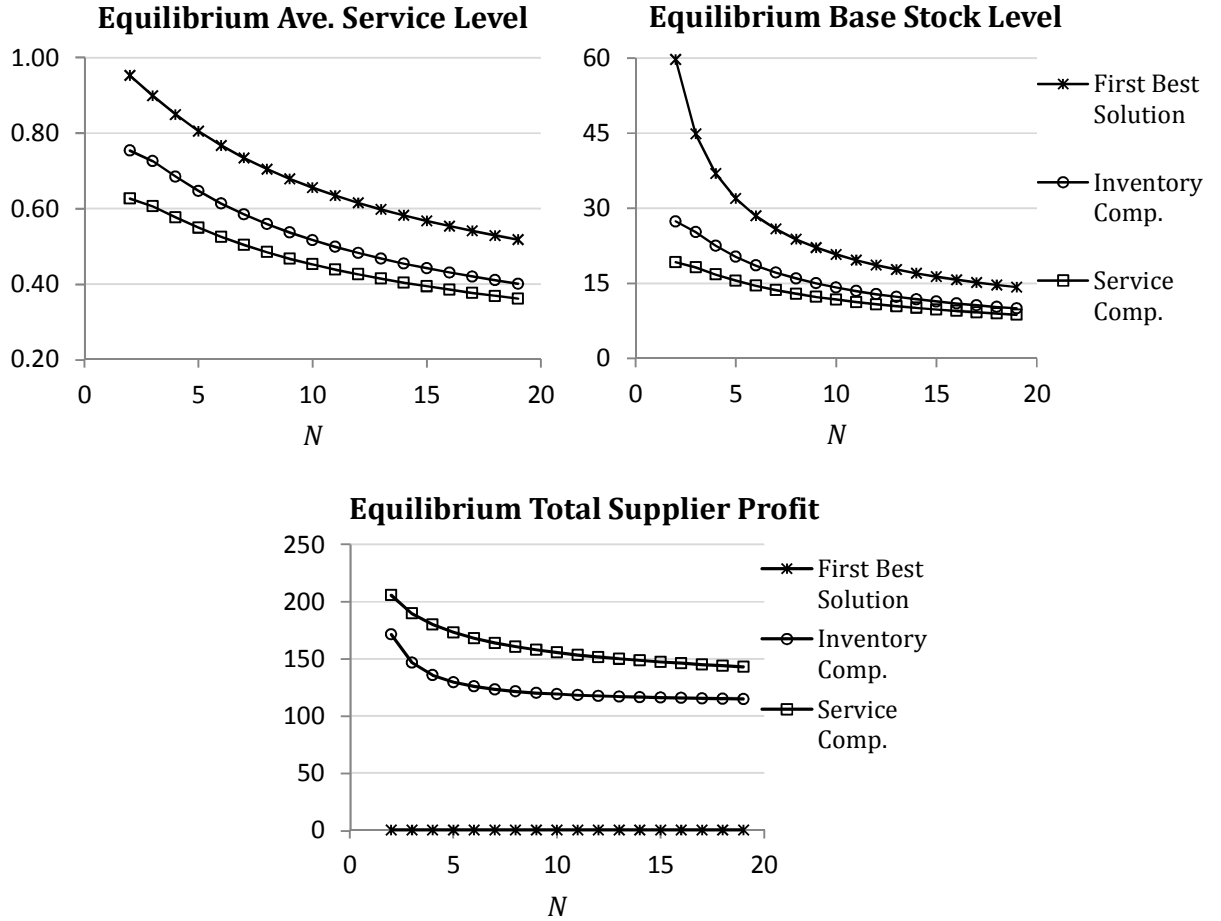


Figure 1 – The impact of the number of suppliers on competition equilibrium  
 $\lambda = 2$ ,  $p = 150$ ,  $c = 20$ ,  $k = 5$ ,  $h = 3$ ,  $\rho = 0.95$

Figure 2 shows the impact of suppliers' utilization. In general, the buyer prefers lower levels of utilization since it results in higher service levels. However, very low utilization levels could result in very high capacity cost for the suppliers and therefore lowers their ability to provide high service levels. This means that at very low utilizations both the buyer and the suppliers are worse off.

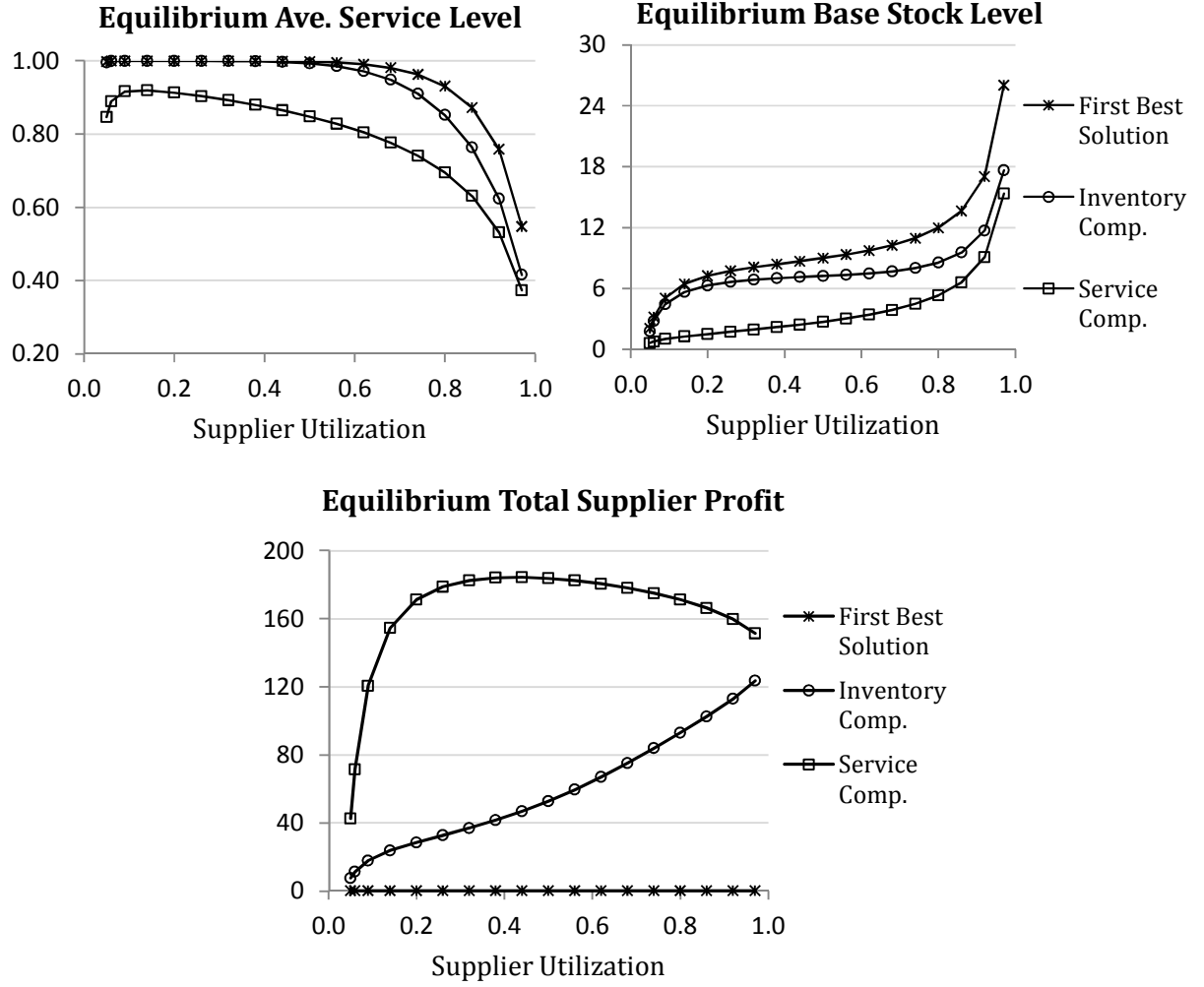


Figure 2 – The impact of supplier utilization on competition equilibrium  
 $\lambda = 2$ ,  $p = 150$ ,  $c = 20$ ,  $k = 5$ ,  $h = 3$ ,  $N = 10$

Although analyzing the impact of the functional form of allocation functions is not the focus of this study, it is worthwhile to have a short discussion in this regard. Theorem 3 states that, under the simple proportional allocation functions, inventory competition always provides higher service levels. Although this result cannot be extended to competitions under all forms of allocation functions, there are more general forms of allocation functions under which we can still prove  $q^I > q^S$ . For instance, consider a more general form of proportional allocation functions. That is,  $\alpha_i^I(z_i, z_{-i}) = z_i^\gamma / \sum_{j=1}^N z_j^\gamma$  and  $\alpha_i^S(s_i, s_{-i}) = s_i^\gamma / \sum_{j=1}^N s_j^\gamma$ , where  $\gamma$  is a positive constant. Using an approach similar to the proof of theorem 3, we can prove that these allocation functions can induce symmetric equilibrium points  $(s_i^* = \hat{s}_i, z_i^* = \hat{z}_i, i = 1, \dots, N)$  such that  $q^I > q^S$ . This more general form of proportional allocation

function is widely used in the competition and contest literature; see Benjaafar et al (2007) and the references therein. Consistent with the results of Cachon & Zhang (2007) and Benjaafar et al (2007), we cannot guarantee the uniqueness of the equilibrium points when  $\gamma > 1$ .

The result that inventory competition provides higher service levels is not guaranteed under all forms of allocation functions. For instance consider the allocation functions  $\alpha_i^I(z_i, z_{-i}) = (1 - a^{z_i}) / \sum_{j=1}^N (1 - a^{z_j})$  and  $\alpha_i^S(s_i, s_{-i}) = (1 - a^{s_i}) / \sum_{j=1}^N (1 - a^{s_j})$ , where  $0 < a < 1$  is a constant. Our numerical results in Figure 3 shows that, using these allocation functions, inventory competition can provide lower service levels when the supplier utilization is in the higher range.

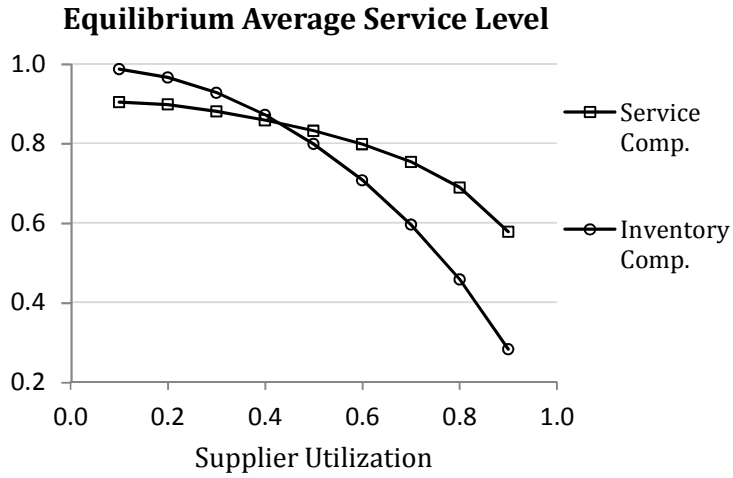


Figure 3 – Competition results under other forms of allocation functions

$$\alpha_i^I(z_i, z_{-i}) = (1 - a^{z_i}) / \sum_{j=1}^N (1 - a^{z_j}) \text{ and } \alpha_i^S(s_i, s_{-i}) = (1 - a^{s_i}) / \sum_{j=1}^N (1 - a^{s_j})$$

$$\lambda = 2, \quad p = 150, \quad c = 20, \quad k = 5, \quad h = 3, \quad N = 10, \quad a = 0.3$$

To be able to focus on the impact of the competition parameter, we continue our analysis using only the simple form of proportional allocation functions as stated in (2) and (3).

## 5. The Impact of Supplier Heterogeneity

To characterize the equilibrium conditions of inventory and service competitions when the suppliers are not identical, we first notice that each supplier's profit function under service competition (5) is concave in the supplier's service level and has a unique positive and finite maximizer (for any set of other suppliers' service levels). Therefore, a Nash equilibrium should be the simultaneous solutions to the first order optimality conditions. Therefore, a Nash equilibrium of the service competition should be the solution to the following system of  $N+1$  equations.

$$\frac{G_s - s_i}{G_s^2} \lambda(p_i - c_i - k_i / \rho_i) = h_i \left( \frac{1}{(1 - s_i) \ln(1 / \rho_i)} - \frac{\rho_i}{1 - \rho_i} \right), \quad i = 1, \dots, N \quad (16)$$

$$G_s = \sum_{i=1}^N s_i \quad (17)$$

Using a similar argument, we can show that a Nash equilibrium of the service competition should be the solution to the following system of  $N+1$  equations.

$$\frac{G_I - z_i}{G_I^2} \lambda(p_i - c_i - k_i / \rho_i) = h_i \left( 1 - \frac{\rho_i^{z_i+1}}{1 - \rho_i} \ln \frac{1}{\rho_i} \right), \quad i = 1, \dots, N \quad (18)$$

$$G_I = \sum_{i=1}^N z_i \quad (19)$$

It is difficult to derive a closed form solution for either of these two systems of equations. We can, however, prove the existence and uniqueness of the solutions which means that each competition mode has a unique Nash equilibrium. Theorem 5 states this result.

**Theorem 5.** *There exists a unique Nash equilibrium for both service and inventory competitions when the allocation functions are defined by (2) and (3), and the suppliers' profit functions are defined by (4) and (5), respectively.*

We use numerical results to provide some insights on the impact of supplier heterogeneity on the competition equilibrium conditions. Figure 4 shows the impact of heterogeneity in suppliers' production cost for the competition between two suppliers. To cover a full range of possibilities, the results in this figure are presented for a relatively wide range of cost ratios. In most cases, however, the difference between suppliers' production costs is smaller than the maximum values shown in this figure. The larger differences between production costs could happen when the suppliers use different production technologies.



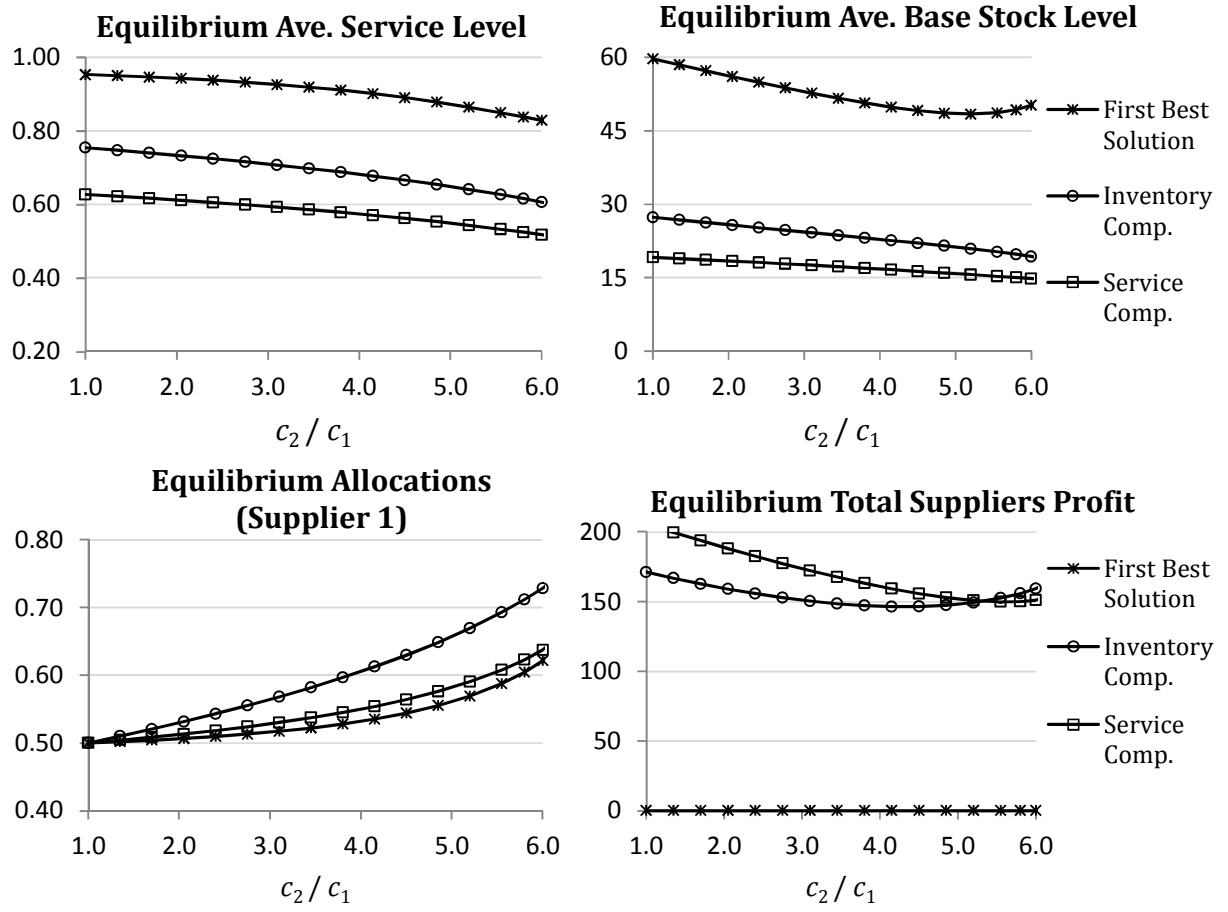


Figure 4 – The impact of heterogeneity in suppliers' production cost on competition equilibrium  
 $\lambda = 2$ ,  $p = 150$ ,  $c_1 = 20$ ,  $k = 5$ ,  $h = 3$ ,  $\rho = 0.95$ ,  $N = 2$

To make the first best solution results comparable with the competition results, we assume that the demand shares of the suppliers are proportional to their service levels in the first best solution. Again, we can see that the inventory competition consistently provides higher service levels. So, it is the buyer's preferred competition mechanism. However, the suppliers' profit is higher under service competition (as was the case for identical suppliers) only for lower levels of heterogeneity. For higher levels of supplier heterogeneity, suppliers' profit under inventory competition can be higher. This is due to the fact that when the difference between suppliers' production costs increases, the profit of the more efficient supplier (the supplier with lower production cost) increases while the profit of the less efficient supplier decreases. As the difference increases both allocation functions (2) and (3) tend to allocate more demand to the more efficient supplier. The inventory competition, however, is more effective in allocating more demand to the supplier with higher efficiency. Therefore, the inventory competition can perform better at

the higher levels of supplier heterogeneity. This means that at this high level of supplier heterogeneity, inventory competition is the preferred mechanism, from both the buyer's perspective and the suppliers' perspective.

It is also interesting to note that optimal competition induces relatively high inventory level (to provide the maximum feasible service levels). For low levels of cost heterogeneity,  $c_2 / c_1$ , the average inventory level under optimal competition is decreasing with heterogeneity, while the demand shares are very close to 0.5 (a nearly balanced allocation). This is because the demand shares are proportional to the service levels while both service levels are close to one. An increase in  $c_2 / c_1$  is equivalent to an increase in  $c_2$  (since  $c_1$  remains fixed), which means that the maximum inventory level of supplier 2 decreases with an increase in  $c_2 / c_1$ . Therefore, considering a nearly balanced allocation at low values of  $c_2 / c_1$ , an increase in heterogeneity means a decrease in the average equilibrium inventory level. For higher values of cost heterogeneity, however, the competition can no longer induce a balanced allocation and the demand share of the supplier with lower production cost grows rapidly (as can be seen in Figure 4). This results in an increase in the average inventory level (since the supplier with a lower cost provides a higher inventory level).

The impact of heterogeneity in price and other cost parameters ( $p_i, k_i$ , and  $h_i$ ) is similar to the impact of heterogeneity in the production cost  $c_i$ . Hence, for the sake of brevity, we do not present them here. The impact of heterogeneity in suppliers' utilization is, however, somehow different. Figure 5 shows this impact for our numerical example.

The numerical results suggest, as the utilization heterogeneity increases, the difference between the average service levels of the two competition types decreases. However, the difference between the suppliers' total profit under the two competition modes increases while service competition consistently provides higher total profit for the suppliers. This is in contrast with the impact of heterogeneity in the production cost. The reason behind this change of behavior is the difference in the behavior of the equilibrium allocations. The demand allocated to supplier 1 (the supplier with higher utilization) is increasing in utilization ratio under inventory competition but decreasing under service competition. The reason behind this observation can intuitively be explained as follows. In Figure 5, to create the utilization heterogeneity, we keep the utilization of supplier 1 constant and reduce the utilization of supplier 2. Lower utilization means that the supplier can replenish the inventory faster. Hence, the inventory level of supplier 2 decreases under both types of competition. As a result, supplier 1, who is competing with

supplier 2, reduces the inventory level as the heterogeneity increases. This results in a decrease in the average inventory level of suppliers under both types of competition (as it can be seen in Figure 4). Our numerical results suggest that the decrease in the inventory level of supplier 1 happens at a slower rate (since it is in response to the decrease in supplier 2's inventory level). Therefore, under inventory competition, supplier 1's demand share increases with the utilization heterogeneity. On the other hand, service level is related to inventory level through supplier's utilization ( $s_i = 1 - \rho_i^{z_i}$ ). Therefore, the service levels under the two types of competition behave differently. Supplier 1's service level decreases due to a decrease in inventory level under both types of competition (utilization is constant). Supplier 2's service level tends to decrease due to a decrease in inventory level too. However, since supplier 2's utilization decreases, numerical results show, the overall impact is an increase in the service level of supplier 2. Therefore, under service competition, supplier 1's demand share decreases with utilization heterogeneity.

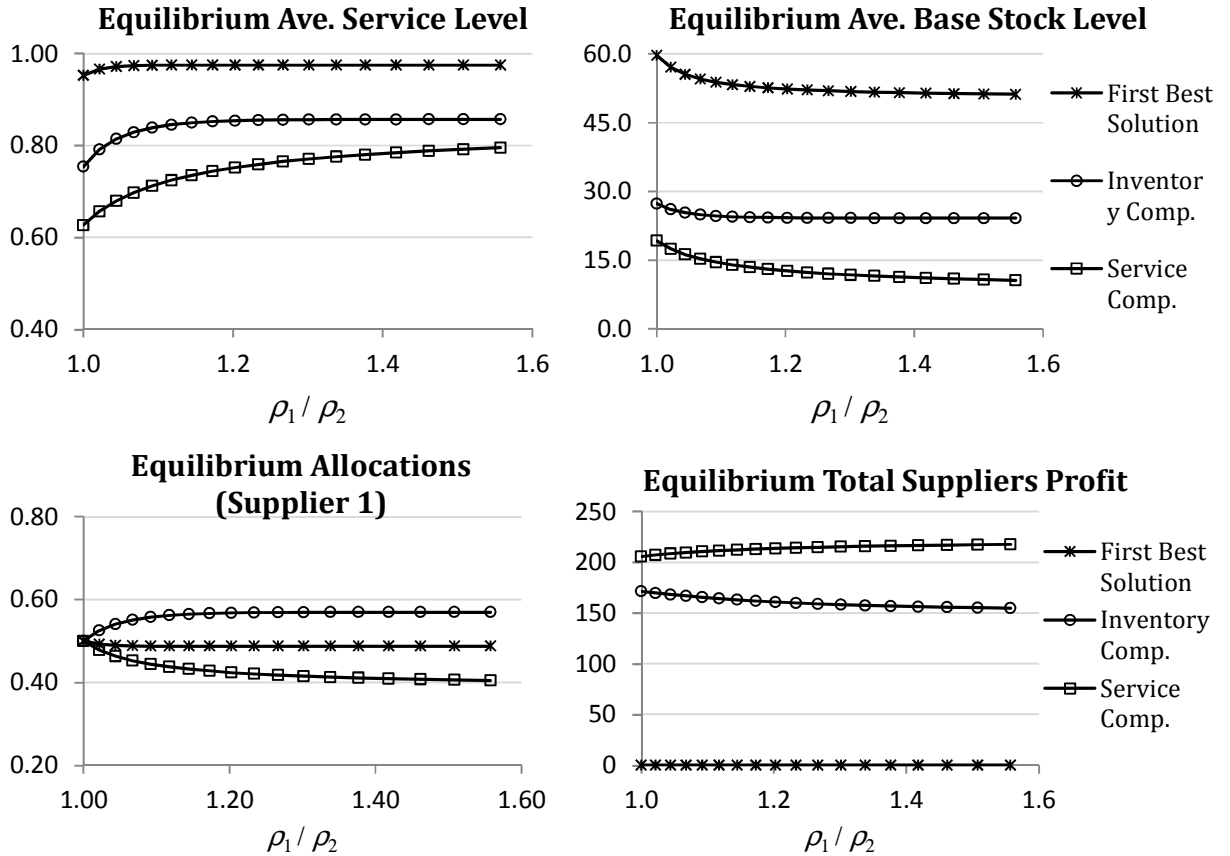


Figure 5 – The impact of heterogeneity in suppliers' utilization on competition equilibrium  
 $\lambda = 2$ ,  $p = 150$ ,  $c = 20$ ,  $k = 5$ ,  $h = 3$ ,  $\rho_1 = 0.95$ ,  $N = 2$

## 6. Optimal Competition Parameter

We observed that the competition parameter (based on which the suppliers compete) can have a significant impact on the intensity of the competition and hence on the outcome for the buyer and the suppliers. We compared two common competition parameters: service (fill rate) and inventory (base stock level). There are other parameters which can be used as competition parameters. Naturally, we expect that each different parameter provides different competition intensity and therefore different outcome for the buyer and the suppliers. For example we can use  $\gamma_i = EB_0 - EB_i$  as the competition parameter, where  $EB_i$  is the expected number of backorders and  $EB_0$  is a benchmark for the expected number of backorders. We know from the  $M/M/1$  queueing results that  $EB_i = \rho^{z_i+1} / (1-\rho)$ . Appendix B shows that the competition based on this parameter has a unique Nash equilibrium. Moreover, when  $EB_0 > \rho / (1-\rho)$ , the average service level under this competition is less than the average service level under service competition.

We can also use a combination of other performance measures to form new competition parameters. In our search for a competition parameter which can induce the highest average service level for the buyer, we designed the following combination of service (fill rate) and inventory (base stock) measures.

$$\xi_i = \frac{1}{N} \left[ \frac{Nh}{\lambda(p-c-k/\rho)} \left( z_i - \frac{\rho}{1-\rho} s_i \right) \right]^{\frac{N}{N-1}}. \quad (20)$$

When we use  $\xi_i$  as the competition parameter, the supplier's profit function can be written as

$$\pi_i^O(\xi_i, \xi_{-i}) = \alpha_i^O(\xi_i, \xi_{-i}) \lambda(p-c-k/\rho) - g^O(\xi_i), \quad (21)$$

where

$$\alpha_i^O(\xi_i, \xi_{-i}) = \frac{\xi_i}{\sum_{j=1}^N \xi_j}, \text{ and} \quad (22)$$

$$g^O(\xi_i) = \frac{\lambda(p-c-k/\rho)}{N} (N\xi_i)^{\frac{N-1}{N}}. \quad (23)$$

The superscript  $O$  indicates that the competition based on  $\xi_i$  can be considered as the *Optimal* competition parameter. This competition parameter is more complicated and less intuitive than direct measures like fill rate, base stock level, or backorders. It is, in fact, an abstract measure that can set the shape of the profit function such that the equilibrium point occurs when each supplier provides its maximum feasible service level. In other words, this parameter can intensify the competition to its maximum level, where each supplier spends all his revenue to provide the maximum feasible level of  $\xi_i$ .

Note, in the definition of this parameter,  $z_i$  and  $s_i$  are interdependent parameters ( $s_i = 1 - \rho^{z_i}$ ). It is not very difficult to show that  $\xi_i$  is an increasing function of either  $z_i$  or  $s_i$ . Therefore, when a supplier guarantees the maximum feasible level of  $\xi_i$ , it means that he guarantees the maximum feasible service level as well. To define this competition parameter, a linear combination of  $z_i$  and  $s_i$  is raised to the power of  $(N-1)/N$  to account for the number of competing suppliers. The linear combination takes into account the cost structure of the suppliers. We can even rewrite the competition parameter in terms of the service cost function (7). That is,  $\xi_i = \left[ Ng^S(s_i) / \lambda(p - c - k / \rho) \right]^{\frac{N-1}{N}} / N$ . The use of supplier cost function in the design of optimal competitions is not unprecedented. Cachon & Zhang (2007) incorporate the supplier capacity cost function to design their linear and proportional allocation functions.

Also note that the cost function stated in (23) is not convex. Hence, the supplier's profit function (21) is no longer concave (as opposed to previous profit functions stated in this paper). Therefore, we need to provide a separate proof for the existence and uniqueness of the Nash equilibrium for this competition. These results are stated in the following theorem. Here, we assume that participating suppliers are required to provide a positive value for the competition parameter,  $\xi_i > 0$ . It is not difficult to verify that a positive  $\xi_i$  is equivalent to a positive service level,  $s_i > 0$ . This means that all participating suppliers are required to provide a positive service level.

**Theorem 6.** *The suppliers' competition based on the allocation function stated in (22) has a unique Nash equilibrium  $\xi^O = (\xi_1^O, \dots, \xi_N^O)$ . The buyer, at this Nash equilibrium, receives the maximum feasible average service level which is equal to the buyer's first best solution,  $q^O = \bar{q} = s^{\max}(1/N)$ . Moreover,  $\pi_i^O(\xi^O) = 0$  and  $\alpha_i^O(\xi^O) = 1/N$ ,  $i = 1, \dots, N$ .*

Since the competition based on  $\xi_i$  (as stated in theorem 6) results in the maximum feasible average service level for the buyer, we can consider it as the optimal competition mechanism. As we can see in theorem 6, the competition equilibrium results in equal demand shares for each supplier. This is possible since the suppliers are identical and the allocation function (22) is symmetric. Therefore, the competition stated in theorem 6 is optimal as long as the suppliers are identical and the buyer is willing to allocate equal demand shares to all suppliers (see our discussion about the optimal set of demand shares in section 3).

However, when the suppliers are heterogeneous or when the buyer is willing to allocate a predefined set of demand shares, the competition stated in theorem 6 cannot achieve the first best solution. To deal with this more complicated setup, the buyer can customize the competition parameter for each supplier.

Consider a setup in which the suppliers are heterogeneous and the buyer is willing to induce a predefined set of demand shares  $\delta = (\delta_1, \dots, \delta_N)$  at the equilibrium of the competition. Here, we assume the set of demand shares chosen by the buyer is feasible. That is,  $\sum_{i=1}^N \delta_i = 1$  and  $0 \leq \delta_i < 1$ ,  $i = 1, \dots, N$ . For this heterogeneous setup, the buyer can design the following competition parameter which is a modification of what we defined in (20).

$$\bar{\xi}_i = \delta_i \left[ \frac{h_i}{\delta_i \lambda (p - c_i - k_i / \rho_i)} \left( z_i - \frac{\rho_i}{1 - \rho_i} s_i \right) \right]^{\frac{1}{1 - \delta_i}}. \quad (24)$$

The allocation function based on this competition parameter has the following form.

$$\bar{\alpha}_i^O(\bar{\xi}_i, \bar{\xi}_{-i}) = \frac{\bar{\xi}_i}{\sum_{j=1}^N \bar{\xi}_j}, \quad (25)$$

Note that (24) tailors the competition parameter for each supplier based on the supplier's cost structure, utilization, and targeted demand share. Following the same approach that we used in the proof of theorem 6, we can prove that the competition between non-identical suppliers, based on allocation function (25), has a unique Nash equilibrium  $\bar{\xi}^O = (\bar{\xi}_1^O, \dots, \bar{\xi}_N^O)$  which results in zero profit for the suppliers and  $\bar{\alpha}_i^O(\bar{\xi}^O) = \delta_i$ . In other words, competition parameter (24) has two important properties. First, it can intensify the competition to a level where all suppliers spend all their revenue to provide the maximum feasible  $\bar{\xi}_i$ , which corresponds to the maximum feasible service level  $s_i^{\max}(\delta_i)$ . Second, at the Nash equilibrium, the allocation function allocates a predefined demand share  $\delta_i$  to each supplier. Therefore, the average service level for the buyer will be the maximum feasible average demand share for the chosen set of demand shares. That is,

$$q^O = \sum_{i=1}^N \delta_i s_i^{\max}(\delta_i).$$

## 7. Conclusion

Through a stylized make-to-stock queueing model, we analyze an outsourcing problem in which the buyer let the suppliers compete for a share of the buyer's demand. The buyer is interested to maximize the average service level that she receives from her suppliers. While the competition literature focuses on the impact of allocation rule on the competition outcome, in this research, we study the impact of *competition parameter* on the intensity of the competition and hence on the service level the buyer receives from her suppliers. The buyer uses a proportional allocation function which can be considered as a simple and intuitive allocation rule. We observed the unexpected result that the competition based on inventory can

induce higher service levels compared to the competition based on the service level itself. We also show that the buyer can design an optimal competition parameter which is a combination of inventory and service levels. This optimal competition parameter can intensify the competition to a level in which each supplier spends all his revenue to provide the maximum feasible service level. When the suppliers are heterogeneous or when the buyer want to allocate a targeted (asymmetric) set of demand shares to the suppliers, the optimal competition parameter should be tailored for each supplier to account for the suppliers' heterogeneous cost structures and/or different targeted demand shares.

Although we derived the results of this paper for a stylized queuing model, we believe the general insights hold for a wider range of applications. That is, the choice of competition parameter can have a significant impact on the competition outcome. Moreover, we should be able to design a competition parameter capable of inducing the maximum level of competition intensity and hence the best result for the buyer.

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## Appendix A

### Proof of Theorem 1

Since  $g^S(s_i)$  is a convex function, it is easy to verify that the supplier's profit function (5) is concave in  $s_i$ . Therefore, to find the Nash equilibrium, it is enough to check the first optimality condition.

$$\frac{\partial \pi_i^S(s_i, s_{-i})}{\partial s_i} = \frac{G - s_i}{G^2} \lambda(p - c - k / \rho) - h \left( \frac{1}{(1 - s_i) \ln(1 / \rho)} - \frac{\rho}{1 - \rho} \right) = 0, \quad i = 1, \dots, N, \text{ where } G = \sum_{i=1}^N s_i$$

We show that this system of  $N$  equations has a symmetric solution  $s_i = s_S^*$ ,  $i = 1, \dots, N$ . Assuming a symmetric solution, we have  $G = N s_S^*$ . We can then rewrite the first optimality condition as

$$\frac{N-1}{N^2 s_S^*} \lambda(p - c - k / \rho) = h \left( \frac{1}{(1 - s_S^*) \ln(1 / \rho)} - \frac{\rho}{1 - \rho} \right).$$

Or equivalently,

$$s_S^* = \left( \frac{N-1}{N^2} \right) \frac{\lambda(p - c - k / \rho)}{h \left( \frac{1}{(1 - s_S^*) \ln(1 / \rho)} - \frac{\rho}{1 - \rho} \right)}.$$

Since the profit function is concave, this equation has a unique solution, which represents a Nash equilibrium  $\mathbf{s}^* = (s_1^*, \dots, s_N^*)$ , where  $s_i^* = s_S^*$ ,  $i = 1, \dots, N$ . The uniqueness of the Nash equilibrium can be concluded from Theorem 1 of Benjaafar et al (2007).

### Proof of Theorem 2

The proof of this theorem is similar to the proof of theorem 1.

### Proof of Theorem 3

We can rewrite equations (14) and (15) as follows

$$\text{Service Competition: } f_s(s_s^*) = \left( \frac{N-1}{N^2} \right) \frac{\lambda(p-c-k/\rho)}{h} \quad (\text{A1})$$

$$\text{Inventory Competition: } f_I(s_I^*) = \left( \frac{N-1}{N^2} \right) \frac{\lambda(p-c-k/\rho)}{h} \quad (\text{A2})$$

Where,

$$f_s(s_s^*) = s_s^* \left[ \frac{1}{(1-s_s^*) \ln(1/\rho)} - \frac{\rho}{1-\rho} \right] \text{ and}$$

$$f_I(s_I^*) = (1-s_I^*) \ln \left( \frac{1}{1-s_I^*} \right) \left[ \frac{1}{(1-s_I^*) \ln(1/\rho)} - \frac{\rho}{1-\rho} \right].$$

Lemma 1 below shows that  $f_s(s) > f_I(s)$  for any  $0 < s < 1$ . Since the left hand sides of equations (A1) and (A2) are constant and equal to each other. We can conclude that the solution to equation (A2) is always greater than the solution to equation (A1). Figure A1 helps to understand this argument.

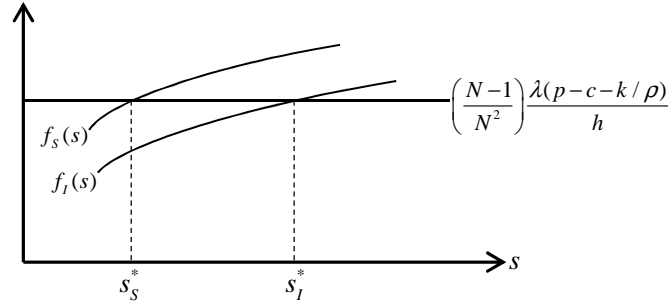


Figure A1 – Optimal service levels for inventory and service competition

**Lemma 1.**  $f_s(s) > f_I(s)$  for any  $0 < s < 1$ .

**Proof of Lemma 1.**

The left brackets in the two functions are the same. Therefore, it is enough to show  $s > (1-s) \ln(1/(1-s))$ . Using the Taylor expansion of the exponential function, it is easy to verify that for any  $x < 1$

$$\frac{1}{x} < e^{\frac{1-x}{x}} = e^{\frac{1-x}{x}}.$$

Therefore,

$$\ln(1/x) < \frac{1-x}{x} \Rightarrow x \ln(1/x) < 1-x.$$

Let  $s = 1-x$ . Then we have

$$(1-s) \ln(1/(1-s)) < s.$$

This concludes the proof of the lemma.

#### Proof of Theorem 4

We prove the theorem for the service competition. The proof for the inventory competition follows the same approach. The right hand side of (A1) is decreasing in  $N$  and approaches zero as  $N$  approaches infinity. It is also easy to verify that  $f_s(0)=0$  and  $f_s(\cdot)$  is an increasing function. Therefore,  $s_s^*$  decreases as  $N$  increases and  $s_s^*$  approaches zero as  $N$  goes to infinity.

#### Proof of Theorem 5

We know that any game with a compact strategy space and a concave utility function has at least one pure strategy Nash equilibrium (See for example Theorem 1.2 in Fudenberg & Tirole, 1991). The suppliers' profit functions under both inventory and service competitions are concave with respect to  $z_i$  and  $s_i$ , respectively. For the service competition, the decision space of each supplier is  $s_i \in [0, \min(1, \bar{s}_i)]$ , where  $\bar{s}_i$  is the unique solution to

$$\lambda(p - c_i - k_i / \rho_i) - g_i^s(\bar{s}_i) = 0$$

Therefore, the strategy space of the service competition is a compact set. A similar argument can be used to show that the strategy space of the inventory game is compact too. This proves the existence of a Nash equilibrium for both competitions.

We next prove the uniqueness of the Nash equilibrium for the service competition. We have already shown that a Nash equilibrium should be a solution to the system of equations (16)-(17). Therefore, for supplier  $i$ , the Nash equilibrium service level should satisfy

$$\frac{G_s - s_i}{G_s^2} \lambda(p_i - c_i - k_i / \rho_i) = h_i \left( \frac{1}{(1 - s_i) \ln(1 / \rho_i)} - \frac{\rho_i}{1 - \rho_i} \right) \quad (\text{A3})$$

For any given value of  $G_s < A$ , equation (A3) has a unique positive solution, where

$$A = \min_i \frac{\lambda(p - c_i - k_i / \rho_i)}{h_i \left[ \frac{1}{\ln(1 / \rho_i)} - \frac{\rho_i}{1 - \rho_i} \right]}. \quad (\text{A4})$$

Since this solution depends on the value of  $G_s$ , we denote it with  $s_i(G_s)$ . The uniqueness of the solution can be concluded from the fact that the right hand side of (A3) is increasing in  $s_i$ , while the left hand side

is decreasing in  $s_i$  and equal to zero at  $s_i = G_s$ . The condition (A4) guarantees that the left hand side is greater than the right hand side at  $s_i = 0$ . Therefore, for a given value of  $G_s < A$ , (A3) has a unique solution. Since we know that this system of equations has at least one solution (we have already proved the existence of the Nash equilibrium), therefore, there is a  $G_s^*$  such that

$$G_s^* = \sum_{i=1}^N s_i(G_s^*). \quad (A5)$$

Now, if we prove that there is only one  $G_s^*$  which satisfies (A5), it means that the system of equations (16)-(17) has a unique solution, which in turn means that there is a unique Nash equilibrium for the service competition. It is easy to verify that  $s_i(G_s)$  is decreasing in  $G_s$ . Therefore, an increase in  $G_s$  means a decrease in the right hand side of (A5) but a decrease in its left hand side. Therefore, we cannot have a  $G_s$  greater than  $G_s^*$  that solves the system of equations. For a similar reason, we cannot have a  $G_s$  smaller than  $G_s^*$  that solves the system of equations. Therefore,  $G_s = G_s^*$  is the only solution. This concludes the proof of the theorem.

### Proof of Theorem 6

Let us write the profit functions of the suppliers in terms of  $y_i = g_i(s_i) = \delta_i \theta_i(s_i)^{1/(1-\delta_i)}$ . That is,

$$\pi_i^O(\xi_i, \xi_{-i}) = \frac{\xi_i}{G} \lambda(p - c - k / \rho) - \frac{\lambda(p - c - k / \rho)}{N} (N \xi_i)^{\frac{N-1}{N}}, \quad (A6)$$

where  $G = \sum_{i=1}^N \xi_i$ . The constraint that all suppliers provide positive  $\xi_i$  means that a Nash equilibrium (if it exists) is an internal point of the decision domain. It is also easy to verify that

$$\lim_{\xi_i \rightarrow \infty} \pi_i^O(\xi_i, \xi_{-i}) \rightarrow -\infty.$$

We also know that  $\pi_i^O(0, \xi_{-i}) = 0$ . Moreover, each supplier's profit function (A6) is decreasing at  $\xi_i = 0$ . This means that each supplier's profit is negative in the vicinity of  $\xi_i = 0$ . Considering the participation condition that each supplier expects a non-negative profit, we can conclude that a Nash equilibrium should satisfy the first order optimality condition. That is, at a Nash equilibrium, for any  $\xi_{-i}$ , we have  $\partial \pi_i^O(\xi_i, \xi_{-i}) / \partial \xi_i = 0$ . Moreover, a Nash equilibrium should correspond to a maximum (not a minimum) point. Therefore, to prove the existence of a unique Nash equilibrium, we need to show that the following system of  $(N+1)$  equations with unknowns  $\xi_i, i = 1, \dots, N$  and  $G$  has a unique solution that maximizes (A6).

$$\frac{\partial \pi_i^O(\xi_i, \xi_{-i})}{\partial \xi_i} = \frac{G - \xi_i}{G^2} \lambda(p - c - k / \rho) - \lambda(p - c - k / \rho) N^{-\frac{1}{N}} \frac{N-1}{N} \xi_i^{-\frac{1}{N}} = 0, \quad i = 1, \dots, N, \text{ and} \quad (A7)$$

$$G = \sum_{i=1}^N \xi_i, \quad (A8)$$

or, equivalently,

$$Y_i^{\frac{1}{N}}(1-Y_i) - \left(\frac{1}{N}\right)^{\frac{1}{N}} \left(1 - \frac{1}{N}\right) G^{1-\frac{1}{N}} = 0, \quad i=1, \dots, N, \text{ and} \quad (\text{A9})$$

$$\sum_{i=1}^N Y_i = 1, \quad (\text{A10})$$

where  $Y_i = \xi_i / G$ . We can rewrite the first order optimality conditions as

$$D(Y_i) = \left(\frac{1}{N}\right)^{\frac{1}{N}} \left(1 - \frac{1}{N}\right) G^{1-\frac{1}{N}}, \quad i=1, \dots, N, \quad \sum_{k=1}^N Y_k = 1, \quad (\text{A11})$$

where  $D(Y_i) \equiv Y_i^{1/N}(1-Y_i)$ . We can see that  $D(0)=D(1)=0$ ,  $D(Y_i) > 0$  for  $0 < Y_i < 1$ , and  $D(Y_i) < 0$  for  $Y_i > 1$ . Also,  $D(Y_i)$  has a maximum at  $1/(1+N)$  which is equal to  $N/(1+N)^{1+1/N}$ . Hence, for any  $G < G^{\max} = (N^{2+1/N} / (N-1)(N+1)^{1+1/N})^{N/(N-1)}$ , equation (A11) has two solutions  $0 < Y_{i,1} < Y_{i,2} < 1$ . We want to argue that  $Y_{i,1}$  corresponds to a local minimum for supplier  $i$ 's profit function, (A6). We observe that the sign of the derivative of the profit function of supplier  $i$ , equation (A7) or (A9), changes from negative to positive when we increase  $Y_i$  from values smaller than  $Y_{i,1}$  to values bigger than  $Y_{i,1}$ . We also observe that, for fixed decisions of the other suppliers,  $Y_i = \xi_i / (\xi_i + \sum_{j \neq i} \xi_j)$  is increasing in  $\xi_i$ . Therefore, when we increase  $\xi_i$ , the sign of equation (A7) changes from negative to positive at  $\xi_i = (Y_{i,1} / (1 - Y_{i,1})) \sum_{j \neq i} \xi_j$ , which in turn means that  $Y_{i,1}$  corresponds to a local minimum of supplier  $i$ 's profit function. Since each supplier tries to maximize his profit,  $Y_{i,1}$  cannot correspond to a Nash equilibrium. Similarly, we can show that  $Y_{i,2}$  corresponds to a local maximum of supplier  $i$ 's profit function. Therefore,  $Y_{i,2}$ ,  $i=1, \dots, N$  is the unique solution to the system of equations (A7) which corresponds to the values of  $\xi_i$  that maximize the profit functions of the suppliers. Figure A2 graphically illustrates the above argument.

For  $G=1$ , the only solution for equations in (A9) that maximizes profit function (A6) is  $Y_{i,2}=1/N$ . It is easy to see that  $Y_{i,2}$  is decreasing in  $G$  (see figure A2). Therefore, for  $G < 1$ , we have  $Y_{i,2} > 1/N$  or equivalently  $\sum_{i=1}^N Y_{i,2} > 1$ , which is not a feasible solution. Also, for  $G > 1$ , we have  $Y_{i,2} < 1/N$  or equivalently  $\sum_{i=1}^N Y_{i,2} < 1$ , which is not a feasible solution as well. Therefore,  $Y_i = 1/N$  and  $G=1$  (or equivalently  $\xi_i^0 = 1/N$ ) is the unique solution of the system of equations (A7)-(A8) which maximizes each profit function in (A6), given all suppliers provide a positive  $\xi_i$ . Replacing from  $\xi_i^0 = 1/N$  in (20), we will have

$$\frac{\lambda(p - c - k / \rho)}{Nh} = z_i^0 - \frac{\rho}{1 - \rho} (1 - \rho^{z_i^0}).$$

Therefore, we can conclude that  $z_i^O = z_i^{\max}(1/N)$ , or equivalently  $s_i^O = s_i^{\max}(1/N)$ . Hence,  $q^O = \bar{q} = s^{\max}(1/N)$ . It is straightforward to verify  $\pi_i^O(\xi^O) = 0$ , and  $\alpha_i^O(\xi^O) = 1/N$ .

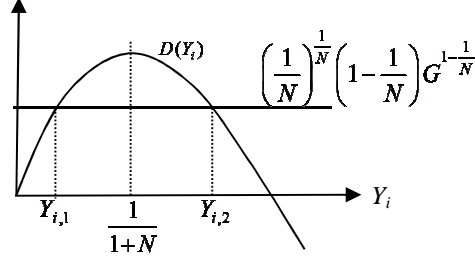


Figure A2 – The solution to first order optimality condition (A9)

## Appendix B

We define  $\gamma_i = EB_0 - EB_i$ . When we use  $\gamma_i$  as the competition parameter, the supplier's profit function can be written as

$$\pi_i^B(\gamma_i, \gamma_{-i}) = \alpha_i^B(\gamma_i, \gamma_{-i}) \lambda(p - c - k/\rho) - g^B(\gamma_i), \quad (\text{B1})$$

where

$$\alpha_i^B(\gamma_i, \gamma_{-i}) = \frac{\gamma_i}{\sum_{j=1}^N \gamma_j}, \text{ and} \quad (\text{B2})$$

$$g^B(\gamma_i) = h \left( \frac{\ln[\rho / [(1-\rho)(EB_0 - \gamma_i)]]}{\ln(1/\rho)} - \frac{\rho}{1-\rho} + EB_0 - \gamma_i \right). \quad (\text{B3})$$

The superscript  $B$  indicates that the competition is based on the expected number of *Backorders*. We will show that the average service level the buyer receives under this competition,  $q^B$ , will depend on the  $EB_0$ .

Since  $g^B(\gamma_i)$  is a convex function, it is easy to verify that the supplier's profit function (B1) is concave in  $\gamma_i$ . Therefore, to find the Nash equilibrium, it is enough to check the first optimality condition.

$$\frac{\partial \pi_i^B(\gamma_i, \gamma_{-i})}{\partial \gamma_i} = \frac{G - \gamma_i}{G^2} \lambda(p - c - k/\rho) - h \left( \frac{1}{(EB_0 - \gamma_i) \ln(1/\rho)} - 1 \right) = 0, \quad i = 1, \dots, N, \text{ where } G = \sum_{i=1}^N \gamma_i$$

We show that this system of  $N$  equations has a symmetric solution  $\gamma_i = \gamma_B^*$ ,  $i = 1, \dots, N$ . Assuming a symmetric solution, we have  $G = N\gamma_B^*$ . We can then rewrite the first optimality condition as

$$\frac{N-1}{N^2 \gamma_B^*} \lambda(p-c-k/\rho) = h \left( \frac{1}{(EB_0 - \gamma_B^*) \ln(1/\rho)} - 1 \right).$$

Since the profit function is concave, this equation has a unique solution, which represents a Nash equilibrium  $\gamma^* = (\gamma_1^*, \dots, \gamma_N^*)$ , where  $\gamma_i^* = \gamma_B^*$ ,  $i = 1, \dots, N$ . The uniqueness of the Nash equilibrium can be concluded from Theorem 1 of Benjaafar et al (2007).

We know  $EB_i = \rho(1-s_i)/(1-\rho)$ . Therefore,  $EB_0 - \gamma_B^* = EB_B^* = \rho(1-s_B^*)/(1-\rho)$ . Hence, we have

$$\frac{N-1}{N^2 \gamma_B^*} \lambda(p-c-k/\rho) = h \frac{1-\rho}{\rho} \left( \frac{1}{(1-s_B^*) \ln(1/\rho)} - \frac{\rho}{1-\rho} \right).$$

Or equivalently,

$$\left[ \frac{1-\rho}{\rho} EB_0 - 1 + s_B^* \right] \left( \frac{1}{(1-s_B^*) \ln(1/\rho)} - \frac{\rho}{1-\rho} \right) = \frac{N-1}{N^2 h} \lambda(p-c-k/\rho). \quad (B4)$$

It is easy to verify that for any  $EB_0 > \rho/(1-\rho)$ , the left hand side of (B4) is larger than the left hand side of (A1). Using the same argument that we used in the proof of theorem 3, we can conclude that  $s_B^* < s_S^*$ .

## Appendix C – Notation

$\alpha_i^S(s_i, s_{-i})$ : Supplier  $i$ 's demand allocation function based on suppliers' service levels

$\alpha_i^I(z_i, z_{-i})$ : Supplier  $i$ 's demand allocation function based on suppliers' base stock levels

$\alpha_i^O(\xi_i, \xi_{-i})$ : Supplier  $i$ 's demand allocation function based on optimal competition parameter

$\alpha_i^B(\gamma_i, \gamma_{-i})$ : Supplier  $i$ 's demand allocation function based on excess number of backorders

$\delta_i$ : The proportion of demand allocated to supplier  $i$

$\gamma_i$ : Excess number of expected backorders beyond the benchmark value

$\mu_i$ : Supplier  $i$ 's processing rate

$\lambda$ : Buyer's demand per unit time

$\pi_i^{FB}(z_i, \delta_i)$ : Supplier  $i$ 's profit function under for the first bust solution

$\pi_i^S(s_i, s_{-i})$ : Supplier  $i$ 's profit function under service competition

$\pi_i^I(z_i, z_{-i})$ : Supplier  $i$ 's profit function under inventory competition

$\pi_i^O(\xi_i, \xi_{-i})$ : Supplier  $i$ 's profit function under optimal parameter competition

$\pi_i^B(\gamma_i, \gamma_{-i})$ : Supplier  $i$ 's profit function under backorder competition

$\rho_i$ : Supplier  $i$ 's utilization

$\xi_i$ : Optimal competition parameter for supplier  $i$

$c_i$ : Supplier  $i$ 's unit production cost

$EB_i$ : Expected number of backorders at supplier  $i$

$EB_0$ : Benchmark expected number of backorders

$g_i^S(s_i)$ : Supplier  $i$ 's holding cost function based on suppliers' service levels

$g_i^I(z_i)$ : Supplier  $i$ 's holding cost function based on suppliers' base stock levels

$g_i^O(\xi_i)$ : Supplier  $i$ 's holding cost function based on optimal competition parameter

$g_i^B(\gamma_i)$ : Supplier  $i$ 's holding cost function based on excess number of backorders

$G_S : \sum_{i=1}^N s_i$

$G_I : \sum_{i=1}^N z_i$

$h_i$ : Supplier  $i$ 's unit holding cost per unit time

$I_i$ : Supplier  $i$ 's actual inventory level

$k_i$ : Supplier  $i$ 's capacity cost per unit of capacity

$N$ : Number of competing suppliers

$p$ : Product unit price (buyer's procurement price)

$q$ : The average service level received by the buyer

$q^S$ : The average service level received by the buyer under service competition

$q^I$ : The average service level received by the buyer under inventory competition

$\hat{q}$ : The maximum feasible average service level received by the buyer (no allocation restriction)

$\bar{q}$ : The maximum feasible average service level received by the buyer (all suppliers receive equal demand shares)

$s_i$ : Supplier  $i$ 's service level

$s_{-i}$ : The vector of service levels provided by the Supplier  $i$ 's competitors

$s_i^{\max}(\delta_i)$ : The maximum service level that a supplier  $i$ 's can provide given his demand allocation is  $\delta_i$

$s_S^*$ : The equilibrium service level under service competition

$\hat{s}_S$ : The equilibrium service level under service competition (general proportional allocation function)

$\mathbf{s}^*$ : The vector of equilibrium service levels under service competition

$z_i$ : Supplier  $i$ 's base stock level

$z_{-i}$ : The vector of base stock levels provided by the Supplier  $i$ 's competitors

$z_i^{\max}(\delta_i)$ : The maximum base stock level that a supplier  $i$ 's can provide given his demand allocation is  $\delta_i$

$z_I^*$ : The equilibrium base stock level under inventory competition



$\hat{z}_I$  : The equilibrium base stock level under inventory competition (general proportional allocation function)

$\mathbf{z}^*$  : The vector of equilibrium base stock levels under inventory competition