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ON THE DECOMPOSITION OF WAGE DIFFERENTIALS

Jeremiah Cotton*

Abstract—The often used method for decomposing wage differentials into human capital and discrimination components is reformulated so that both the disadvantage, or "cost," discrimination imposes on a black or minority wage earner and the advantage, or "benefit," it bestows on a white or majority wage earner can be estimated.

Introduction

THE theories of human capital investment and economic discrimination taken together suggest that differences in the average wages of racial groups occur both because of differences in their average skills or productivity characteristics and differences in the way the market treats or evaluates membership in a particular group, the level of skills notwithstanding. Moreover, according to the dual labor market hypothesis the chief way the market is able to maintain and perpetuate such treatment differences is by routing minorities in disproportionate numbers into the secondary sector of the labor market and mainly white males into the primary sector.

There have been a number of empirical studies in which attempts have been made to decompose observed racial wage and earnings differentials into these hypothesized "skill" and "treatment" components. One of the most often used decomposition methods was first employed in demography by Kitagawa (1955) and later popularized in the sociology literature by Duncan (1968) and Althauser and Wigler (1972), and in the economics literature by Oaxaca (1973) and Blinder (1973). In his seminal work on labor market discrimination, Becker (1971) defined a competitive market discrimination coefficient for labor of different productivity as the difference between their observed wage ratio and the wage ratio that would prevail in the absence of discrimination. Oaxaca (1973) expressed this difference in percentage terms as:

\[ D = \frac{\bar{W}^w/\bar{W}^b - MP^w/MP^b}{MP^w/MP^b} \]  

(1)

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1 Oaxaca investigated sexual rather than racial wage differences. However, inasmuch as the subsequent analysis focuses where \( \bar{W}^w/\bar{W}^b \) is the observed white–black average wage ratio, and \( MP^w/MP^b \) is the ratio of the white–black average marginal products, which by assumption is the average wage ratio in the absence of discrimination. Expressed in logarithmic form, (1) becomes the white–black average wage differential:

\[ \ln \bar{W}^w - \ln \bar{W}^b = \ln MP^w - \ln MP^b + \ln(D + 1). \]  

(2)

The difference between the marginal products, \( \ln MP^w - \ln MP^b \), is part of the wage differential that is due to differences in white and black productivity, and \( \ln(D + 1) \) is the treatment, or discrimination component.

Now, in general, \( \ln \bar{W} \) can be estimated by

\[ \ln \bar{W} = \sum_{j=0}^{K} B_j \bar{X}_j, \]

where the \( \bar{X}_j \)'s are average productivity-determining characteristics, and the \( B_j \)'s are least-square regression coefficients. Thus, (2) can be written as

\[ \ln \bar{W}^w - \ln \bar{W}^b = \sum_{j=0}^{K} B_j^w \bar{X}_j^w - \sum_{j=0}^{K} B_j^b \bar{X}_j^b \]  

(3)

With some elementary manipulations the terms on the right-hand side of (3) can be decomposed into either

\[ \ln \bar{W}^w - \ln \bar{W}^b = \sum_{j=0}^{K} B_j^w (\bar{X}_j^w - \bar{X}_j^b) + \sum_{j=0}^{K} \bar{X}_j^w (B_j^w - B_j^b) \]  

(4)

or

\[ \ln \bar{W}^w - \ln \bar{W}^b = \sum_{j=0}^{K} B_j^w (\bar{X}_j^w - \bar{X}_j^b) + \sum_{j=0}^{K} \bar{X}_j^b (B_j^w - B_j^b). \]  

(5)

The first terms on the right-hand sides of (4) and (5) are estimates of \( \ln MP^w - \ln MP^b \), and the second terms are estimates of \( \ln(D + 1) \). Oaxaca explained these alternative forms of the decomposition in the following manner:

On the basis of either of two assumptions, we can estimate the white–black wage ratio that would exist in the absence of discrimination: If there were no discrimination, 1) the wage structure currently faced by blacks would also apply to whites; 2) the wage structure currently faced by whites would also apply to blacks. Assumption one (two) says that blacks (whites) on racial wage differences I have taken the liberty of substituting "white" for "male" and "black" for "female" in the reproductions of his formulas and the quote from his text.
would on average receive in the absence of discrimination the same wages as they presently receive, but that discrimination takes the form of whites (blacks) receiving more (less) than a nondiscriminating labor market would award them. (Oaxaca, 1973, p. 895)

The formulations in (4) and (5) correspond to Oaxaca's first and second assumptions, respectively. In his empirical work he treated the issue as essentially an index number problem and obtained estimates from both formulations, using them to establish the range within which the true values of the components presumably would fall. Some subsequent decompositioners followed Oaxaca's example of estimating both forms, while others opted for one form or the other, or some variant of both. Several analysts who used (5) did so because they believed the wage structure that would prevail in the absence of discrimination was more likely to be close to the white wage function than to the wage function of blacks. 3

So far the principal concern of those who have considered this procedure has been largely statistical in nature. Among some of the adherents the problem of omitted variables has been the main worry. Since the second component is a residual, for it to be an exact measure of labor market discrimination all of the factors that determine the wage must be present and properly accounted for. If they are not, if perhaps because of data limitations some have been excluded and others poorly measured, then the residual will reflect these omitted influences as well, and will therefore either over- or underestimate the extent of discrimination. 4 This is a long-standing problem and at present nothing very much can be done except to recognize the check it places on the interpretation of results.

The criticism, however, that comes closest to revealing the main flaw in the construction of the Oaxaca decomposition is that made by Butler (1982). His most telling argument is that the attempt to measure labor market discrimination by differences in white–black regression coefficients confounds market, or demand-side sources of discrimination with those that originate on the non-market, or supply-side. Such coefficients are taken


\[
\ln \bar{W}^w - \ln \bar{W}^b = \sum_{j=1}^{G} B_j^*(\bar{x}_j^w - \bar{x}_j^b) + \sum_{j=1}^{G} (\bar{x}_j^w - \bar{x}_j^b)(B_j^* - B_j^b). \tag{2.1}
\]

This, however, necessitated the addition of a third term, called the "interaction" term. Masters interpreted this term as a measure of the relative magnitude of discrimination against blacks who have above-average educational and other productivity skills. See also Althauser and Wigler (1972) and Iams and Thornton (1975).

3 See, e.g., Gwartney and Long (1978) and Cotton (1985). Curiously, Masters took the same approach in defense of the second component of his decomposition (see footnote 2 above), arguing that the alternative form, \( \sum \bar{x}_j^b(B_j^w - B_j^w) \) supposed that "we would give whites the black earnings function without changing that function. This assumption is less realistic than assuming no change in the white earnings function when it is given to blacks because they are only a small percentage of the population." (Masters, 1974, p. 343, footnote 3). Yet in his choice of his second term he apparently neglected his own good advice.

4 Suppose data are available on only \( G \) of the \( K \) productivity characteristics that are assumed to fully determine the wage. Then the racial wage equation for the \( i \)th individual is

\[
\ln W_i^w = \sum_{j=0}^{G} B_j^i X_j + \epsilon_i^r.
\]

where

\[
\epsilon_i^r = \sum_{j=G+1}^{K} B_j^i X_j^i.
\]

The effect of the omitted characteristics are captured in the intercept. The decomposition of the wage differential now takes the form:

\[
\ln \bar{W}^w - \ln \bar{W}^b = \sum_{j=1}^{G} B_j^*(\bar{x}_j^w - \bar{x}_j^b) + \sum_{j=1}^{G} (\bar{x}_j^w - \bar{x}_j^b)(B_j^* - B_j^b)
+ \sum_{j=1}^{G} \bar{x}_j^b(B_j^w - B_j^b) + (B_0^w - B_0^b).
\]

where \( B_0^w = B_0^w + \bar{m}^w \). The last two terms of the sum comprise the residual component and Jones (1983) has shown that it is impossible to assign a unique value to either element of this component. Thus, the residual is a mixture of discrimination and other omitted influences. However, it must be noted that many of these omitted factors, e.g., school quality or family background, may be themselves the result of past discrimination and to control for them may be tantamount to controlling for significant sources of discrimination.
from reduced-form equations and are therefore an
amalgam of both demand and supply structural
coefficients. And because of past supply-side dis-

rimination in the provision of education and
other skill-acquiring opportunities, the demand for
black labor might be more elastic than the de-

mand for the more capital-compatible white labor
even in the absence of discrimination. In which
case even though blacks and whites are identical
in all other respects the white $B_j$'s will be larger
than the black $B_j$'s and any measure of dis-

crimination based on their differences will be
overstated.

Butler is correct in questioning the comparisons
of black and white regression coefficients. He is
not correct, however, to assume that these are the
coefficients that would prevail in the absence of
discrimination. For without discrimination we
would not expect differences in the black and
white $B_j$'s to persist. Perhaps in the short run just
after discrimination has been eliminated one might
observe blacks and whites with different average
skills because of different opportunities in the
past, but in the long run as blacks are assured of
competing on equal terms in the same markets as
whites the differences in supply characteristics can
be expected to diminish along with differences in
demand for black and white labor. Indeed it is
this very expectation of continuing convergence of
black and white skills that is heralded by the
proponents of the vintage hypothesis.⁵

In this paper it is contended that the Oaxaca
decomposition procedure is flawed because of its
failure to portray adequately the most critical of
Becker's original conditions, viz., the wage struc-
ture that would prevail in the absence of
discrimination. If Oaxaca's first assumption were
used, i.e., if we assumed that blacks would receive
the same wage in the absence as in the presence of
discrimination, then barring envy or malice to-
wards white wage earners (and unrequited benevo-

\[ X_t(B_t - B^*) \] 

dence toward employers), blacks would have no
particular economic reason for desiring an end to
such discrimination since their own wages would not be
affected thereby.

Separately considered, each assumption ab-

\[ \sum B_j^* (X_j^w - X_j^b) \] 

tracts from the central reality of wage and other
forms of economic discrimination: not only is the
group discriminated against undervalued, but the
preferred group is overvalued, and the underval-

\[ \sum B_j^* (X_j^w - X_j^b) = \sum X_j^w (B_j^* - B_j^*) \] 

tation of the one subsidizes the overvaluation of
the other.⁶ Thus, the white and black wage struc-
tures are both functions of discrimination and we
would not expect either to prevail in the absence of
discrimination.

The Nondiscriminatory Decomposition

The derivation of a more suitable decomposi-
tion formula starts with Becker's assumption that
in the absence of discrimination in perfectly com-
petitive markets whites and blacks would be per-
fect substitutes in production. Or put another way,
in the absence of discrimination the only reason
wage differences would arise would be because of
differences in productivity characteristics. There-
fore, in the absence of discrimination the wage
structures are assumed to be equal: $B^{*w} = B^{*b} =
B^*$, where $B^*$ is the nondiscriminatory wage struc-
ture.

Now consider the hypothetical term, $\sum B_j^* (X_j^w
-
X_j^b)$. This is the difference in the current white
and black average productivity characteristics
evaluated as the market would in the absence
of discrimination. It is therefore the "true" value of
the skill component of the wage differential.

Consider also the hypothetical term, $\sum B_j^* (X_j^w
-
X_j^b)$. These are the current white average productivity
characters valued as they would be in the
absence of discrimination. The difference between
this term and the first term on the right-hand side
of (3) is solely due to differences in the way whites
are currently treated and the way they would be
treated in the absence of discrimination,

\[ \sum B_j^* (X_j^w - X_j^b) = \sum X_j^w (B_j^* - B_j^*) \] 

⁵ Some Marxists claim that the only beneficiaries of dis-

\[ \sum B_j^* (X_j^w - X_j^b) \] 

crimination are capitalist employers and that both white
and black workers lose or are "undervalued." (See Reich, 1968.)

\[ \sum B_j^* (X_j^w - X_j^b) \] 

And while in general agreement, Harris (1978) suggests that
perhaps the rate of exploitation of black labor exceeds that of
white labor. Others such as Baron (1975) and Baran and
Sweezy (1966) appear to believe that once capitalism passed
its competitive to its monopoly stage the profitable use of
discrimination diminished for the capitalist class. Now ide-
ological and psychological factors rather than pecuniary ones
account for the persistence of racism.
This is therefore that part of the treatment component of the wage differential which, if positive, is due to whites’ “pure” treatment advantage. A similar situation exists with respect to blacks, and we have

$$\sum B_j^* \bar{X}_j^b - \sum B_j^b \bar{X}_j^b = \sum \bar{X}_j^b (B_j^* - B_j^b).$$

This is that part of the treatment component which, if positive, measures blacks’ “pure” treatment disadvantage. The average wage differential is therefore decomposed as

$$\ln \bar{W}^w - \ln \bar{W}^b = \sum B_j^* (\bar{X}_j^w - \bar{X}_j^b) + \sum \bar{X}_j^w (B_j^w - B_j^*) + \sum \bar{X}_j^b (B_j^* - B_j^b). \tag{6}$$

In this decomposition the treatment or discrimination component is made up of two elements, one representing the amount by which white productivity characteristics are overvalued (the “benefit” of being a white worker), and the other the amount by which black productivity characteristics are undervalued (the “cost” of being a black worker).^7

A graphic distinction between the Oaxaca decomposition and the formulation in (6) is shown in figure 1. As drawn, the three simplified wage functions are assumed to depend solely on productivity characteristic $\bar{X}$. The Oaxaca decomposition of the wage differential is $(B^w \bar{X}^w - B^w \bar{X}^b) + (B^b \bar{X}^b - B^b \bar{X}^b)$. This is the equation (5) version of the decomposition and it overestimates the “true” productivity difference and underestimates the “true” treatment difference. The decomposition given in (6) is $(B^w \bar{X}^w - B^w \bar{X}^w) + (B^b \bar{X}^b - B^b \bar{X}^b) + (B^w \bar{X}^w - B^w \bar{X}^w) + (B^b \bar{X}^b - B^b \bar{X}^b)$.

The major operational weakness of (6), however, is the fact that the $B^*$ vector is unobserved and therefore must be estimated if the formulation is to be useful for empirical work. Such an estimator of course is subject to criticism inasmuch as its construction must be based on a number of rather strong assumptions about the nature of $B^*$. The

first of these assumptions is a restatement of the conclusions previously drawn about the expected outcomes of wage and other forms of economic discrimination, viz., in the absence of discrimination whites would receive a lower average wage than they currently receive and blacks would receive a higher average wage. Thus,

$$\sum B_j^w \bar{X}_j > \sum B_j^* \bar{X}_j > \sum B_j^b \bar{X}_j.$$

Second, it is assumed that in the absence of discrimination the prevailing market structure will be some function of the forces that currently determine the white and black wage structures. This assumption is simplified by specifying $B^*$ as a linear function of $B^w$ and $B^b$, the respective white and black wage structures. Third, it is assumed that the nondiscriminatory wage structure will be closer to the current white wage structure than to the current black wage structure.$^8$ This third assumption is operationalized by weighting the white and black wage structures by the respective proportions of white and black males in the employed civilian male labor force.$^9$ Thus, the estimator of

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^7 The interpretation of, say, $\sum B^* \bar{X}^b$ is no more difficult than the interpretation of the Oaxaca term, $\sum B^* \bar{X}$. The former values the black characteristics “as if there were no discrimination” and the latter values the black characteristics “as if blacks had the white wage structure.” Nor for that matter is it any more difficult than the interpretation of $B$, the regression coefficient, as the change in the dependent variable “if a unit change occurred in the independent variable”—a most hypothetical event indeed.

^8 Bergmann (1971, p. 310) concluded that “for the great majority of whites the end of discrimination would have only a minor effect on rates of pay. Those whites in the lowest bracket would bear the brunt of the change.”

^9 Some decompositioners who recognized that the nondiscriminatory wage structure should lie between the wage structures of the majority group and that of the minority group arbitrarily assigned equal weight to both groups (see Coleman et al., 1971; Lopreato and Poston, 1977; and Reimers, 1983). Reimers wrote a general expression for the skill component of the wage differential in matrix form as

$$X^w - X^b) (D B^w + (1 - D) B^b)\tag{9.1}$$

where $I$ is the identity matrix and $D$ a diagonal matrix of
A-59. The relative proportion of white to black males involved in the labor force would result in a redistribution of income but no real gain in actual output."

In order to compare the results from the non-discriminatory formula in (6) with the alternative formulations of the Oaxaca decomposition given in (4) and (5), each will be used to estimate the components of the wage differential of white and black males.

Empirical Wage Model and Decomposition Results

The data used in this analysis were taken from the 1% sample of the Public Use Samples of the 1980 Census. Coverage was restricted to white and black males 16 years and over who had positive earnings and hours worked and who resided in the Northeastern states of New York and Pennsylvania; the Midwestern states of Illinois, Indiana, Michigan, Ohio and Wisconsin; the Southwestern states of California and Texas; and the Southern states of Georgia and North Carolina. There were 21,341 white males and 2,785 black males in the samples.

The mean values of the explanatory variables of the wage model along with their respective regression coefficients are given in table 1. The comparative performance of the two most important variables in the model, education and work experience, was generally as expected. White males had about a year and a half more schooling than blacks and the average rate of return to an additional year of schooling for whites was considerably greater than that for blacks. In addition, the wage–experience profile of white males was well above the black male profile.

The log hourly wage for white males was 2.0125, and the corresponding geometric mean wage was $7.48. For blacks the log wage was 1.7987, and the geometric mean wage was $6.04. Thus, the resulting log wage differential was 0.2138, and the mean wage difference was $1.44.

The decomposition of these differentials using equation (6), the hypothetical formulation that incorporates an estimate of the nondiscriminatory wage structure, is given in the first row of table 2. There it is estimated that approximately 49% of the log wage difference was due to white males' skill or productivity advantage evaluated as it would have been in the absence of discrimination. Translated into dollars and cents it means that about 71¢ of the $1.44 wage gap was due to skill differences between whites and blacks. The white male treatment advantage accounted for some $7.48 \times 0.49 = 3.6688$ of the total wage gap. This is the difference in the current white male wage and the wage they would receive if there were no discrimination.

The treatment disadvantage component for black males was over 28% of the log wage difference and represented about 41¢ of the $1.44 wage gap. This is the difference in the current black male wage and the wage they would receive if there were no discrimination.

In the last two rows of table 2 estimates of the skill and treatment components obtained by using the Oaxaca decomposition formulas of equations (4) and (5) are presented. As expected, equation (4) underestimates the "true" value of the skill differential and overestimates the treatment component, whereas equation (5) does the reverse.
TABLE 1.—MEAN VALUES AND REGRESSION COEFFICIENTS OF MODEL VARIABLES
(ESTIMATED STANDARD ERRORS IN PARENTHESES)

<table>
<thead>
<tr>
<th>Variables</th>
<th>White</th>
<th>Black</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Values</td>
<td>Regression Coefficients</td>
</tr>
<tr>
<td>Education</td>
<td>12.13</td>
<td>0.0756* (0.0112)</td>
</tr>
<tr>
<td>Work Experience</td>
<td>16.44</td>
<td>0.0377* (0.0086)</td>
</tr>
<tr>
<td>Work Experience²</td>
<td>468.86</td>
<td>-0.0006* (0.00002)</td>
</tr>
<tr>
<td>Married, Wife Present</td>
<td>0.768</td>
<td>0.2969* (0.0255)</td>
</tr>
<tr>
<td>Once Married</td>
<td>0.086</td>
<td>0.1147* (0.031)</td>
</tr>
<tr>
<td>Urban Residence</td>
<td>0.411</td>
<td>0.1219* (0.0099)</td>
</tr>
<tr>
<td>Veteran</td>
<td>0.538</td>
<td>0.0239 (0.0211)</td>
</tr>
<tr>
<td>Government Worker</td>
<td>0.196</td>
<td>-0.0276* (0.0131)</td>
</tr>
<tr>
<td>Region:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northeast</td>
<td>0.324</td>
<td>0.0638b (0.0392)</td>
</tr>
<tr>
<td>Southwest</td>
<td>0.244</td>
<td>0.0955 (0.0665)</td>
</tr>
<tr>
<td>South</td>
<td>0.098</td>
<td>-0.1052* (0.0412)</td>
</tr>
<tr>
<td>Industry:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td>0.102</td>
<td>0.2891* (0.0488)</td>
</tr>
<tr>
<td>Mfg. Durables,/Nondurables</td>
<td>0.341</td>
<td>0.1966* (0.0475)</td>
</tr>
<tr>
<td>Trans &amp; Pub Util.</td>
<td>0.123</td>
<td>0.2311* (0.049)</td>
</tr>
<tr>
<td>Wholesale/Retail Trade</td>
<td>0.141</td>
<td>-0.0267 (0.0444)</td>
</tr>
<tr>
<td>Insurance/Real Estate</td>
<td>0.057</td>
<td>0.1582* (0.0617)</td>
</tr>
<tr>
<td>Household Service</td>
<td>0.0007</td>
<td>0.0953 (0.1395)</td>
</tr>
<tr>
<td>Miscellaneous Service</td>
<td>0.117</td>
<td>-0.1249* (0.0423)</td>
</tr>
<tr>
<td>Public Administration</td>
<td>0.083</td>
<td>0.1169* (0.0378)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.0</td>
<td>0.3133* (0.0527)</td>
</tr>
</tbody>
</table>

R² 0.281
SSE 0.4057
N 21,341
In Hourly Wage 20125
Mean Hourly Wage $7.48
(Geometric Mean)

*Significant at the 0.01 level or less
bSignificant at the 0.05 level or less

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Indeed, had we only these two equations with which to capture the values of the white-black skill and treatment components we would have a very wide range to deal with. According to equation (4), only about 5% of the wage differential was due to skill differences and the other 95% to treatment differences. On the other hand, equation (5) estimated that nearly 52% of the differential was due to a skill gap and about 48% to differential treatment.

**Conclusion**

The main defect in previous attempts to decompose wage differentials has been shown to be due to a failure of past decompositioners to appreciate fully the underlying theory of discrimination that should have guided both the construction of their decomposition formulas and the interpretation of the resulting components. As a consequence they ended up either underestimating or overestimating the hypothesized skill and treatment differences.

The form of the decomposition procedure derived in this paper not only yields more nearly accurate estimates of the components of the wage differential but also models the true state of differential treatment by estimating the "cost" to the group discriminated against as well as the "benefits" accruing to the favored group.

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