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# Outsourcing to Non-Identical Suppliers via Service Competition

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## 1 Introduction

Outsourcing has emerged as a major trend in many manufacturing and service industries. Although early outsourcing decisions were based on cost, they are increasingly being based on the quality of service promised by potential suppliers. In fact, the weak bargaining position of suppliers in many industries means that the buyer sets the price, with quality of service being a primary differentiator among suppliers. Large retailers, such as Wal-Mart, and manufacturers, such as Dell, have developed sophisticated methods for tracking and rewarding the quality of service of their suppliers, their third party logistics providers, and other business process contractors. Quality of service, in these and other industries, is usually measured in terms of the availability of the demanded good or service at the time it is requested. For physical goods, typical measures of service quality, or service levels, include fill rate, expected order delay, the probability that order delay does not exceed a quoted lead-time, and the percentage of orders fulfilled accurately. For services, measures of service level include expected customer waiting time, the probability that the customer receives service within a specified time window, and the probability that a customer does not leave (renege) before being served. Selecting suppliers who are able to consistently deliver on one or more of these service measures is particularly important when the buyer envisions a long term relationship with her suppliers.

In this paper, we consider a single buyer who wishes to outsource a fixed demand for a manufactured good or service at a fixed price to a set of  $N$  suppliers. We examine the value of competition as a mechanism for the buyer to elicit good service quality from her suppliers. In particular, we consider a scheme in which the buyer allocates a proportion of demand to each supplier, with the proportion a supplier receives increasing in the service level she offers. Suppliers compete for expected market share, which increases in the offered service level.

The suppliers affect their service levels by exerting *effort* once they receive a positive portion of demand, with the cost of effort increasing in the service level offered and the demand allocated. Each supplier chooses a service level to maximize her own expected profit, subject to the behavior of other competing suppliers. In making this decision, the supplier effectively weighs the market share benefits of each service level against its associated cost.

The possibility of inducing service quality through competition raises several important questions. For example, under what conditions does service competition lead to an equilibrium? How does the number and type of suppliers affect the buyer's service quality and the suppliers' expected profits? Is it more desirable for the buyer to contract with suppliers that are equally efficient or to have a mix of suppliers with varying capabilities? How should the buyer choose parameters for the competition to maximize the quality of service she receives? In particular, what is the impact of the allocation functions on the buyer's quality of service and is it possible for the buyer to choose an allocation function that forces the suppliers to provide the maximum feasible service level? In this paper, we address these and other related questions.

## 2 Competition Formulation and Nash Equilibrium

We consider a system with a single buyer that seeks to outsource the provisioning of a product with an expected demand quantity  $\lambda$  to  $N$  potential suppliers. The price of the product,  $p$ , is fixed and identical across all suppliers. However, suppliers may differ in the service level they offer to the buyer and the unit cost they incur in producing or servicing the product. Let  $s_i \geq 0$  denote the service level offered by supplier  $i$ ,  $\lambda_i = \alpha_i \lambda$  the amount of demand allocated to supplier  $i$ ,  $0 \leq \alpha_i \leq 1$ ,  $c_i$  her unit production cost, and  $r_i = p - c_i$  her unit revenue for all  $i = 1, \dots, N$ . Also, let  $f_i(s_i, \lambda_i)$  denote the cost supplier  $i$  incurs in providing service level  $s_i$  ( $s_i \geq 0$ ) if given demand allocation  $\lambda_i$ , with  $f_i(s_i, \lambda_i)$  non-decreasing in both  $s_i$  and  $\lambda_i$  and  $f_i(0, \lambda_i) = 0$ . We choose to separate production costs from service level costs since we assume that unit production costs remain the same regardless of the service level offered. We assume that each

supplier commits to producing and delivering to the buyer the amount of demand allocated while maintaining the service level promised. Table 1 lists possible interpretations for service level and the corresponding costs.

Table 1 - Examples of measures of service level and associated costs

| Service measures ( $s_i$ )                                                                                                                                                                                                                 | Associated service costs ( $f_i(s_i, \lambda_i)$ )                                                                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>• Fill-rate</li> <li>• Expected delay</li> <li>• Expected backlog level</li> <li>• Reliability in meeting a quoted lead-time</li> <li>• Yield rate</li> <li>• Order fulfillment accuracy</li> </ul> | <ul style="list-style-type: none"> <li>• Inventory investment &amp; maintenance costs</li> <li>• Capacity investment &amp; maintenance costs</li> <li>• Lead-time reduction costs</li> <li>• Product quality improvement costs</li> <li>• Workforce training costs</li> </ul> |

The buyer announces a criterion for allocating demand among the suppliers with the understanding that a supplier  $i$  can increase her fraction of demand by increasing the service level she promises to offer the buyer. This does not preclude the buyer from taking into account factors other than quality of service or from reflecting preferences for certain suppliers. We assume that, once promised, service levels offered by the suppliers are enforceable. In practice, this would occur if the cost or, more likely, the associated effort expended by each supplier after the buyer allocates demand, is observable. The buyer can then ascertain whether or not a supplier has exerted sufficient effort (expended sufficient cost) to meet the promised service level. For instance, the buyer may observe the amount of capacity invested by the supplier after the demand was allocated and determines whether or not it is sufficient to meet the expected leadtime that was initially promised by the supplier. Of course, there can also be settings where suppliers voluntarily deliver on promised service levels (regardless of observability of cost or effort) because they worry about their reputation or expect repeated interactions with the buyer in the future.

Demand allocation is carried out via a demand allocation function vector  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_N)$  where  $\alpha_i(s_i, s_{-i})$  specifies the fraction of demand allocated to supplier  $i$  given the supplier's own service level  $s_i$  as well as the service levels  $s_{-i} = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_N)$  offered by her competitors with  $0 \leq \alpha_i(s_i, s_{-i}) \leq 1$ . The function  $\alpha_i(s_i, s_{-i})$  is nondecreasing in  $s_i$  and equal to zero when  $s_i = 0$ , for  $i = 1, \dots, N$ .

The expected quality of service received by the buyer is then

$$q(\mathbf{s}) = \sum_{i=1}^N \alpha_i(s_i, s_{-i}) s_i, \quad (1)$$

where  $\mathbf{s} = (s_1, \dots, s_N)$ . The buyer chooses a structure for  $\alpha^C$  to induce high quality of service by rewarding better performing suppliers with either higher market share (under SA) or a higher probability of selection (under SS). Given the buyer's choice of  $C$  and  $\alpha^C$ , the suppliers respond by competing against each other for the buyer's fixed demand.

Each supplier competes by choosing a service level  $s_i$  that maximizes her own expected profit, subject to the behavior of other suppliers. Under SA competition, this implies supplier  $i$  will choose  $s_i$  to maximize

$$\pi_i(s_i, s_{-i}) = \alpha_i(s_i, s_{-i})\lambda r_i - f_i(s_i, \alpha_i(s_i, s_{-i})\lambda), \quad (2)$$

Note that supplier  $i$ 's *expected* revenue and *expected* cost depend on her own service level  $s_i$  as well as the service level profile  $s_{-i}$  of her competitors. We assume that the contractual promises of the suppliers regarding service level are enforceable once chosen. We also assume all parties have full access to information about each other's costs. In systems where some of the parameters are random variables, all suppliers are assumed to be risk neutral and profit maximizers. Costs are incurred by a supplier only after demand allocations are announced by the buyer and only if the supplier receives a positive portion of demand. The following theorem lays out sufficient conditions for an Equilibrium to exist; proofs of all results can be found in Benjaafar et al. (2005) and Elahi et al. (2006).

**Theorem 1:** A Nash equilibrium  $(s_1^*, s_2^*, \dots, s_N^*)$  exists if (a)  $\alpha_i$  is continuous in  $\mathbf{s}$  and non-decreasing concave in  $s_i$ ,  $\alpha_i(s_i^*, s_{-i}^*) \leq 1$ , and (b)  $f_i$  is increasing and convex in  $s_i$ , for  $i = 1, \dots, N$ .

It is difficult to make additional statements about the Nash equilibrium without further specifying the service cost or allocation functions. Therefore, we shall consider special cases where the analysis is tractable and yields useful managerial insights. We first consider the simple case of a linear function of effort cost  $f_i(s_i, \alpha_i^{SA}(s_i, s_{-i})\lambda) = k_i s_i$  and a service-proportional allocation function  $\alpha_i(s_i, s_{-i}) = s_i / \sum_{j=1}^N s_j$ . The expected profit of supplier  $i$  can then be expressed as  $\pi_i(s_i, s_{-i}) = \alpha_i(s_i, s_{-i})\lambda r_i - k_i s_i$ . Without loss of generality, we assume that  $k_1/r_1 \leq k_2/r_2 \leq \dots \leq k_N/r_N$ . Also, we let  $M$  refer the largest integer in such that

$$\frac{k_M}{r_M} < \frac{M}{M-1} Q_M, \quad (3)$$

where  $Q_M = \frac{\sum_{j=1}^M k_j/r_j}{M}$ . We use the ratio  $k_j/r_j$  to describe the efficiency of supplier  $j$ , with lower ratios corresponding to higher efficiency.

**Theorem 2:** *Given a linear cost function and a service-proportional allocation function, there exists a unique Nash equilibrium such that the service levels and allocations at the Nash equilibrium are as follows*

$$s_i^* = \begin{cases} \frac{M-1}{M^2} \frac{\lambda}{Q_M} \left( M - \frac{k_i}{r_i} \frac{M-1}{Q_M} \right) & \text{if } i \leq M \\ 0 & \text{if } i > M \end{cases}, \quad \text{and} \quad \alpha_i^* = \begin{cases} 1 - \frac{M-1}{M^2} \frac{k_i/r_i}{Q_M} & \text{if } i \leq M \\ 0 & \text{if } i > M \end{cases}. \quad (4)$$

The above theorem (a version was first discussed in Stein (2002)) states that only  $M$  from the pool of  $N$ ,  $M \leq N$ , suppliers offer a positive service level and are, therefore, allocated positive demand. These suppliers are the  $M$  most efficient ones (i.e., suppliers with the  $M$  lowest ratios  $r_i/k_i$ ). Not surprisingly, the more efficient the supplier the higher the service level it offers and the higher the market share it receives. In the case of equally efficient suppliers ( $r_i/k_i = r_j/k_j$  for all  $i \neq j$ ), all  $N$  suppliers offer the same service level and receive an equal fraction of demand equal to  $1/N$ . The expected quality of service received by the buyer is given by

$$q^* = \frac{M-1}{M^2} \frac{\lambda}{Q_M} \left[ 1 + (M-1)^2 \frac{\sigma_M^2}{Q_M^2} \right] \quad (5)$$

where  $\sigma_M^2$  is the standard deviation of the ratios  $k_i/r_i$ ,  $i=1, \dots, M$ . Hence,  $q^*$  is sensitive to both the mean and the variance of the suppliers' efficiency. Perhaps surprisingly,  $q^*$  is increasing in as  $\sigma_M^2$  for fixed  $M$  and  $Q_M$ , implying that asymmetry in the efficiency of the suppliers can be beneficial to the buyer.

The effect of the number of participating suppliers is more subtle. Increasing the number of potential suppliers  $N$  can be either beneficial or detrimental to service quality, or without effect. In general adding efficient suppliers is beneficial, especially if it leads to a reduction in  $M$ . Adding suppliers who are equally efficient to existing ones can be detrimental to service quality while less efficient supplier may not have effect if those suppliers end up being allocated no demand.

### 3 Orchestrating Supplier Competitions

In orchestrating a supplier competition, a buyer may choose the initial number of participating suppliers, the price to pay each supplier, and the type of demand allocation function. It turns out that allocation functions can be a powerful tool at the disposal of the buyer, which can be effective in inducing suppliers to offer higher service levels and in some cases the maximum feasible service level. In the following theorem, we characterize an allocation function that would indeed maximize quality of service to the buyer (i.e., a buyer-optimal allocation function).

**Theorem 3:** *The following allocation function is optimal:*

$$\alpha_i(s_i, s_{-i}) = \begin{cases} \theta_i s_i^{\beta_i \gamma} - \frac{1}{\hat{N}} \left( \theta_i \sum_{j=1}^{\hat{N}} s_j^{\beta_i \gamma} - 1 \right) & \text{if } i \leq \hat{N} \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where,  $\hat{N} \leq N$  is the largest integer such that  $s_i > 0$  and  $\alpha_i > 0$ ,

$$\theta_i = \frac{\hat{N}}{\hat{N} - 1} \frac{a_i (\hat{N} Q_{\hat{N}})^{\beta_i \gamma - 1}}{\beta_i \gamma \lambda^{\beta_i \gamma}} \quad \text{for } i = 1, \dots, \hat{N};$$

$$\beta_1 = 1; \quad \beta_i = \frac{\hat{N}}{t_i \hat{N} + \gamma(\hat{N} - 1)(1 - t_i)} \quad \text{for } i = 2, \dots, \hat{N}; \quad \gamma \leq \min_i \frac{\hat{N} t_i}{1 + (\hat{N} - 1)t_i},$$

$$a_i = k_i / r_i \text{ and } t_i = a_1 / a_i.$$

A version of the above allocation function was first introduced by Cachon and Zhang (2004) for a system with identical suppliers. In that setting, it can be shown that this allocation function induces each supplier to provide the maximum feasible service (a service level that leads to zero supplier profit).

## 4 Extensions

It is possible to extend the analysis to consider more general cost functions where cost may depend in a non-linear fashion on service level and may also depend on the amount of demand allocated. It is also possible to treat alternative forms of competition where, for example, a single supplier is selected and allocated the entire demand. The supplier selection is still subject to a competition where service level determines the likelihood that a particular supplier is chosen as the sole supplier.

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