10-1-2011

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Abstract

We investigate how a competition can be designed to maximize expected profit for a buyer who wishes to allocate demand among a diverse set of suppliers when his profit is dependent on the supplier's service levels. The candidate suppliers are heterogeneous in their capacities and cost structures, and compete for shares of the buyer's demand based on their promised service levels. To characterize the optimal competition, we first identify a family of allocation functions that are *service maximizing,* meaning they can intensify the competition to a point where each supplier provides its maximum feasible service level and the outcome of the competition is a predefined set of demand shares. We show that using a service maximizing allocation function is a necessary condition for solving the buyer's problem. We then characterize the optimal demand allocation set and, when they are endogenous, the optimal procurement prices. When both demand allocation and procurement prices can be chosen by the buyer, we find that the competition also maximizes supply chain profit. Through a set of numerical examples, we show that the benefit of using this optimal competition design, including its specified demand allocation function and suggested procurement prices, can be significant.

October 2011

1. Introduction

While some outsourcing and procurement decisions are still driven primarily by price, the service capability of suppliers is becoming an increasingly important factor for many manufacturing and service firms. A recent industry survey reveals that the most important factor in outsourcing decisions is process efficiency and quality, with cost reduction ranked third (Mazars Annual Outsourcing Survey 2010). Large retailers, such as Wal-Mart, and manufacturers, such as Dell, place a premium on the service levels their suppliers provide and use sophisticated supplier rating systems for tracking and rewarding supplier performance. Service quality features prominently in the supplier rating systems used by other firms as well. For example, Pratt & Whitney, a major manufacturer of aircraft engines, industrial gas turbines and space propulsion systems, considers only quality rating (60%) and delivery rating (40%) measures in its supplier performance index[1]. Saturn Electronics, a global supplier to original equipment manufacturers (OEMs), also uses a rating system that weighs on-time delivery (20%) and quality compliance (30%) more significantly than cost (15%)[2].

This increased focus on service level is driven, in part, by the availability in many industries of multiple qualified suppliers. The relatively weak bargaining position of these suppliers, particularly when the buying firm is large, allows the buying firm to set the price, with service level becoming a primary factor in differentiating between suppliers. Concern about service level is also driven by the operational policies adopted by many firms, which emphasize on-demand production and on-time delivery. Such policies make firms particularly vulnerable to poor supplier performance because of the limited safety stocks and safety lead-times these firms maintain, with quality of service from the suppliers directly affecting their revenue. High supplier service level is, of course, critical to firms that have chosen to compete on the basis of customer service, or that can extract a price premium for higher service. In some cases, the suppliers deal directly with the buying firm's customers, such as in call centers. In those cases, the service level the suppliers provide directly affects the service levels end customers receive.

Service level is typically measured in terms of the availability of the demanded good or service at the time it is requested. For physical goods, typical measures of service quality include fill rate, expected order delay, the probability that order delay does not exceed a quoted lead-time, and the percentage of orders fulfilled within specification. For services, service level measures include expected customer

[1] http://www.rocketdynetech.com/supplierinfo/documents/spi.pdf
[2] http://www.saturnee.com/uploads/supplier_rating_criteria.pdf

waiting time, the probability that the customer receives service within the specified time window, and the probability that a customer does not renege before being served.

While the importance of having suppliers provide high service levels is clear, it is less clear how firms should go about inducing their suppliers to invest in service quality, particularly when suppliers vary in their capacity levels and cost structures. In settings where the buyer also has the power to set the procurement price, it is not clear how these prices should be set to entice high service levels without compromising profit. Similarly, when the buyer has the flexibility to allocate demand among more than one supplier, it is not clear how such allocation should be carried out to induce maximum service quality and how these allocations are affected by the procurement prices. One approach is to simply select those suppliers that promise to offer the highest service level and accept the lowest price. However, when suppliers vary in their capabilities, and these capabilities are common knowledge among all parties, there may not be enough incentive for the more capable suppliers to maintain their maximum feasible service levels. Another alternative is to negotiate an incentive contract with a subset of the suppliers where, for example, suppliers are financially rewarded (penalized) for providing service levels higher (lower) than a negotiated service level. However, the outcome of such negotiation can be unpredictable or prone to renegotiations.

In this paper, we explore a third alternative where a firm (the buyer) stages a competition among a set of potential suppliers and allocates a fraction of demand to two or more suppliers based on the service level each promises to provide. The fraction of demand allocated to each supplier is calculated using a scoring function, which we call the *allocation function*. The buyer chooses this allocation function and announces it to the suppliers. The suppliers respond by quoting a service level, to which they promise to commit should they be allocated any fraction of the buyer's demand. In determining a service level to quote, each supplier maximizes his own profit, which is a function of the fraction of demand he receives and the cost of service level he provides. The buyer's revenue is affected by the service levels her suppliers provide, since it affects the quality of service the buyer provides for her customers in return. The buyer's objective is to design the allocation function (and set the procurement prices if she has the power to do so) so as to maximize her expected profit.

To design an optimal competition mechanism that results in maximum profit for the buyer, we provide a general framework for characterizing optimal allocation functions. We introduce the notion of *service-maximizing* allocation and show that this property provides a necessary condition for the optimality of a solution to buyer's problem. A service-maximizing allocation function is one that induces suppliers to provide the maximum feasible service level for the amount of demand allocated. We then identify a family of proportional allocation functions that are service-maximizing. We characterize the optimal competition design under conditions of (1) exogenous and (2) endogenous procurement prices. In the second case, the buyer chooses both the allocation function and procurement prices. In settings where it is desirable to fix the proportion of demand each supplier receives, we also show that it is possible to design the allocation function to induce these desired proportions as an outcome of the competition.

The supplier competition described here is similar to the *SA competition* discussed in Benjaafar et al. (2007). However, that paper is not concerned with determining optimal allocation functions. Instead, the focus is on studying the behavior of suppliers who are engaged in a supplier competition orchestrated by a single buyer under a specific service-proportional allocation, exogenously determined. The analysis is limited to identical suppliers with identical costs and revenue structures and with no constraints on capacity. Also in their case, procurement prices are exogenously determined and the buyer measures the performance of the procurement mechanism through a (demand-weighted) average service level. In this paper, we evaluate the performance of designed mechanisms through a more direct measure: buyer's profit. We also consider a more general supply structure, with suppliers who are heterogeneous in their costs and capacity levels.

In addition to Benjaafar et al. (2007), the other paper most related to our competition setting is Cachon and Zhang (2007). They consider a specific context where suppliers are modeled as single server queues and compete in terms of investment in service rates. Higher service rates translate into higher service levels in the form of lower queueing delays for the buyer. Similar to Benjaafar et al. (2007), they treat the case of homogeneous suppliers with identical revenue and cost structures. They compare different demand allocations and show that a *linear* allocation function leads suppliers to invest in the maximum feasible service rates for the fraction of demand they are allocated. However, because they consider only symmetric allocations (suppliers that provide the same service rates are allocated the same amount of demand), the proposed allocation does not necessarily maximize overall quality-of-service. As with Benjaafar et al. they do not consider capacity limits, and procurement price selection.

A general review of the literature on service-based supplier selection and procurement is included in Benjaafar et al. (2007). For the sake of brevity, we will not reproduce it here. For more recent papers see Jin and Ryan (2009), Xiaoyuan Lu et al (2009), and Zhou and Ren (2010). However, we should note that much of the existing literature has focused on schemes involving competition among identical suppliers or competition involving specific allocation functions, typically proportional allocation functions. Few results exist for settings with heterogeneous suppliers or suppliers with capacity constraints. We are not aware of any results on the joint optimization of demand allocation and procurement prices.

There is also related literature in economics on rent-seeking contests. In a rent-seeking contest, there are N contestants who compete for a prize. The probability that a contestant wins the prize (the rent) increases with his expenditures and decreases in the expenditures of other contestants. A review of important results from this literature can be found in Mueller (2003, Ch. 15), Congleton et al. (2008), and Konrad (2009). A focus of this literature is on documenting the so-called inefficiency of rent-seeking contests. Rent-seeking is viewed as wasteful since the total expenditures by the contestants can equal the value of the prize itself, a phenomenon called *rent dissipation.* However, in systems with non-identical contestants, it has been shown that there may not be complete rent dissipation; see for example Hilman and Reily (1989), Paul and Wilhite (1990), Nti (1999), and Dixit (1987). That is, heterogeneity in the contestants' characteristics can diminish the intensity of the competition. In our research, we show that by carefully designing an allocation function (the equivalent of a selection probability in the rent seeking context), one may induce the contestants to exert the maximum feasible effort even when they have non-identical characteristics.

The rest of the paper is organized as follows. Section 2 describes our problem setting and formulation. Section 3 focuses on the optimal form of competition when the procurement prices are set exogenously. Section 4 discusses the structure of the optimal solution. Section 5 extends these results for cases where the buyer also has the power to set the procurement prices. Section 6 presents a series of numerical examples, highlighting the benefits of the optimal competition mechanism. Section 7 offers concluding remarks.

2. Model Formulation and Structure of the Buyer's Problem

Our supply chain consists of a buyer and N potential suppliers who differ in their production and service capabilities. The buyer wishes to allocate her total demand, λ, across these suppliers in a manner that maximizes profit.[3] We assume the buyer's revenue depends directly on the service level offered by the suppliers. The buyer sets a demand allocation scheme (i.e., a scoring rule) through which suppliers

[3] Demand can be interpreted as either a single quantity covering one sales period or a demand-rate maintained over multiple periods.

compete for higher demand shares based on the service level they choose to offer. We first assume the procurement price, p, is constant across suppliers, and so not a factor in the buyer's allocation decision. We will relax this assumption in section 5.

The competition proceeds as follows. First, the buyer announces a criterion for allocating demand across suppliers with the property that the fraction of demand a supplier receives is increasing in the service level he offers. More specifically, the buyer carries out the allocation through a demand allocation function vector $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)$ where α_i is a function $\alpha_i(s_i, s_{-i})$ specifying the fraction of demand allocated to supplier i given the supplier's announced service level s_i *and* the service levels $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_N)$ offered by his competitors with $0 \leq \alpha_i(s_i, s_{-i}) \leq 1$. We assume the function $\alpha_i(s_i, s_{-i})$ is non-decreasing in s_i and equal to zero when $s_i = 0$, for $i = 1, \ldots, N$.

In determining the optimal demand allocation, the buyer must evaluate and compare the suppliers' capabilities[4]. Each supplier is uniquely characterized by its capacity level, ω_i, unit operating cost, c_i, and service-related costs, $f_i(s_i, \alpha_i(s_i, s_{-i}))$, i=1,..., N. We assume that the supplier set is large enough to cover demand, i.e., $\sum \omega_i \geq \lambda$. We also assume that supplier i's service related cost is a function of the proportion of demand allocated to the supplier as well as the supplier's service level. We focus on a particular class of plausible service cost functions of the form

$$f_i(s_i, \alpha_i(s_i, s_{-i})) = k_i \lambda \alpha_i(s_i, s_{-i}) + v_i(s_i), \tag{1}$$

where k_i is a positive constant and $v_i(s_i)$ is a continuous, increasing and convex function in s_i, with $v_i(0) = 0$, for i=1,..., N. We assume that $v_i(s_i)$ is twice differentiable. This function generalizes service cost functions used in prior research (e.g., Benjaafar et al. 2007, Cachon and Zhang 2007) by accounting for supplier heterogeneity. The first term, $k_i \lambda \alpha_i(s_i, s_{-i})$, is the *demand-dependent* service cost, which varies linearly with the demand allocated to the supplier. The second term, $v_i(s_i)$, captures supplier-specific costs that increase only with the service level itself. This *demand-independent* cost is not affected by the amount of demand allocated[5]. Examples of service related costs that fit this model include

[4] The setting we consider is one where there is full information regarding the cost structure of the suppliers. This would be the case when the primary cost drivers of the suppliers are in the public domain (e.g., settings where costs are determined by regional factors such as labor costs; taxes and regulations; cost of materials and energy; type of production technology used; and transportation costs). The assumption of full information builds on assumptions made previously in the literature and serves as an upper bound on performance for the buyer and the supply chain. It also provides insight into the design of procurement mechanisms if the buyer is to leverage knowledge of cost and capacity differences among the supplier and to assess the benefit derived from this knowledge.

[5] A more general form of $f_i(s_i, \alpha_i(s_i, s_{-i}))$ can be defined in which the demand-dependent cost is also a function of the service level. That is, $f_i(s_i, \alpha_i(s_i, s_{-i})) = \alpha_i(s_i, s_{-i}) u_i(s_i) + v_i(s_i)$, in which $u_i(s_i)$ is a continuous, non-decreasing, and convex function of s_i. Our main results also hold for this more general form of the cost function.

investments in capacity, inventory, transportation, and/or continuous improvement efforts. We will elaborate on one specific example in section 6, as part of our numerical study.

Each supplier i responds to the announced allocation function, **α,** by choosing a service level that maximizes his expected profit, $\pi_i(s_i, s_{-i})$, subject to the behavior of other suppliers. In choosing their service levels, suppliers trade off the potential revenue benefit of a higher service level against the cost of providing this service.

The supplier's profit under this competition setting can now be stated as

$$\pi_i^S(s_i, s_{-i}) = \alpha_i(s_i, s_{-i})\lambda(p - c_i - k_i) + v_i(s_i). \quad (2)$$

Note that profit depends on a supplier's own service level s_i as well as the service level profile s_{-i} of his competitors. In keeping with previous studies, we assume service levels are enforceable, all suppliers are risk neutral, and costs are incurred only if the supplier receives a positive portion of demand (i.e., $\lambda\alpha_i(s_i, s_{-i}) > 0$). We also assume that the reservation profits of the suppliers are zero.

The buyer's profit is determined by the revenue received from her customers minus the procurement prices paid to her suppliers to cover their production and service costs. We assume the buyer is directly rewarded (penalized) for high (low) service quality through the revenue she receives from her own customers. More specifically, we characterize the buyer's reward (penalty) as the sum of increasing concave functions of the service levels provided by each supplier, $h(s_i), i = 1, \cdots, N$, weighted by the proportion of demand receiving that service. Therefore, the buyer's revenue can be written as

$$r(\mathbf{s}, \alpha(\mathbf{s})) = p_B\lambda + \sum_{i=1}^{N} \alpha_i(\mathbf{s})\lambda h(s_i), \quad (3)$$

where p_B is the buyer's unit selling price. Because the revenue function is concave in all elements of $\mathbf{s} = (s_1, ..., s_N)$, it has the appealing property of decreasing returns to service. This revenue structure captures situations where the buyer's customers observe the service level provided by the suppliers while the buyer either collects a premium for high service or is penalized for poor service. Applications where the suppliers' service level is observable by end customers include outsourcing after-sales services, call centers, roadside assistance, or drop shipping when the supplier directly ships to the end customer[6].

The buyer's profit function can then be written as

$$\pi^B(\mathbf{s}, \alpha(\mathbf{s})) = r(\mathbf{s}, \alpha(\mathbf{s})) - \sum_{i=1}^{N} (\alpha_i(\mathbf{s})\lambda p) = r(\mathbf{s}, \alpha(\mathbf{s})) - p\lambda. \quad (4)$$

We can now state the buyer's problem under this competition setting as

[6] For example, when make-to-order suppliers directly ship to the buyer's customers and service level is measured by the probability of meeting a quoted lead time, we have $h(s_i) = -a(1 - s_i)$, where a is the penalty paid for each unit delivered later than the quoted lead time.

$$\max_{(\alpha_1(\mathbf{s}),\ldots,\alpha_N(\mathbf{s}))\in\mathscr{F}^N} \pi^B(\mathbf{s},\boldsymbol{\alpha}(\mathbf{s})) = r(\mathbf{s},\boldsymbol{\alpha}(\mathbf{s})) - p\lambda, \quad (5)$$

subject to:

$$s_i = \arg\max_x \left(\alpha_i(x, s_{-i})\lambda[p - c_i - k_i] - v_i(x)\right), \; i = 1,\ldots,N \quad (6)$$

$$\pi_i^S(s_i, s_{-i}) = \alpha_i(s_i, s_{-i})\lambda\left[p - c_i - k_i\right] - v_i(s_i) \geq 0, \; i = 1,\ldots,N, \quad (7)$$

$$\alpha_i(\mathbf{s})\lambda \leq \omega_i, \; i = 1,\ldots,N \quad (8)$$

where $\mathscr{F}^N = \{(\alpha_1(\mathbf{x}),\ldots,\alpha_N(\mathbf{x}))\}$ is the set of all N-dimensional vectors of functions with $\alpha_i : \mathbf{R}^N \mapsto [0,1]$ and $\sum_{i=1}^N \alpha_i(\mathbf{x}) = 1$. Note that the optimization is carried out over all vectors of functions in $\mathscr{F}^N$. The first set of constraints (6) reflects the suppliers' subgame, where each supplier chooses a service level to maximize his own profit for a given set of service levels chosen by his competitors. Each supplier's decision in this subgame is affected not only by his competitors' decisions, but also by the form of allocation function set by the buyer. Constraint (7) guarantees non-negative profits for suppliers. Constraints (8) define capacity limits on the allocation function values. Since in this problem we seek optimal forms of functions rather than optimal values, we cannot solve the problem through regular optimization approaches. Instead, we use an indirect approach as outlined in the next section.

3. Optimal Service-Based Competition

To solve the buyer's problem, we use a two-step process. In the first step, we show how the buyer can design an allocation function that induces the suppliers to invest in their maximum feasible service levels for a given vector of demand allocations $\boldsymbol{\delta} = (\delta_1, .., \delta_N)$, where δ_i denotes the fraction of demand assigned to supplier i, $0 \leq \delta_i \leq 1$ and $\sum_{i=1}^N \delta_i = 1$. This implies the demand allocated to supplier i is $\lambda_i = \lambda\delta_i$. In the second step, we show how the buyer can choose a set of demand allocation values that maximize her expected profit subject to the dynamics of the competition.

Step 1: Finding a Service-Maximizing Allocation Function

Our goal in this step is to characterize an allocation function that maximizes the buyer's profit for a given demand allocation vector $\boldsymbol{\delta}$. In other words, we seek an allocation function which maximizes the buyer's profit while achieving a set of predefined demand shares across suppliers at the competition equilibrium (i.e., $\alpha_i(s_i^*, s_{-i}^*) = \delta_i$, where (s_i^*, s_{-i}^*) is the equilibrium service level vector). Throughout our analysis, we assume the given target demand shares are feasible, implying that $\sum_{i=1}^N \delta_i = 1$ and $\delta_i\lambda \leq \omega_i$ for $i = 1,\ldots,N$.

We begin by defining a specific type of allocation function, which we refer to as *service-maximizing.* When $\boldsymbol{\delta}$ is fixed, the buyer's profit increases in each supplier's service level and so the buyer's problem reduces to finding an allocation function which induces the maximum feasible service levels. In other words, we are looking for an allocation function that intensifies the competition to a point where each supplier chooses a service level that zeroes out his expected profit (i.e., leaves him with his reservation profit level). Let $s_i^{\max}(\delta_i)$ denote this maximum service level associated with δ_i, i = 1, …, N. At this service level, each supplier applies all the revenue gained from his associated allocation (i.e., $\delta_i \lambda(p - c_i - k_i)$) to cover his demand independent service cost $v_i(s_i^{\max}(\delta_i))$. We are now ready to define the service-maximizing property.

Definition 1: *An allocation function* $\alpha_i(s_i, s_{-i})$ *is service-maximizing, with respect to* $\boldsymbol{\delta}$*, if it induces a Nash equilibrium service level vector* $\mathbf{s}^* = (s_1^*, \ldots, s_N^*)$ *for which* $s_i^* = s_i^{\max}(\delta_i)$ *and* $\alpha_i(s_i^*, s_{-i}^*) = \delta_i,\ i = 1, \ldots, N.$

This property is important because if an allocation function is shown to be service-maximizing, this ensures that it maximizes the buyer's profit when $\boldsymbol{\delta}$ is given. It is a sufficient condition for optimality in this special case.

In our search for a service maximizing allocation function, we focus on proportional allocation functions, since they are commonly used in the literature. In particular, we consider the following general characterization that allows for heterogeneity across suppliers:

$$\alpha_i(s_i, s_{-i}) = \frac{g_i(s_i)}{\sum_{j=1}^{N} g_j(s_j)}, \tag{9}$$

for i = 1, …, N, where $g_i(s_i)$ is a non-decreasing function of s_i with $g_i(0) = 0$. Unlike prior literature, which focuses almost exclusively on symmetric functions[7] (e.g., Benjaafar et al. 2007, Cachon and Zhang 2007, Allon and Federgruen 2005, and the references therein), we allow the parameters of the allocation function to differ by supplier. Symmetric functions are a subset of this family, with the restriction that $g_i(s_i) = g(s_i)$ for i = 1, …, N.

We know from prior research that a proportional allocation function does not guarantee a unique Nash equilibrium solution. For example, Benjaafar et al. (2007) show that a symmetric proportional allocation function in a system with identical suppliers does not guarantee uniqueness of the Nash equilibrium when $g(s_i)$ is not concave (see also Cachon and Zhang (2007) for a similar result). It follows

[7] Symmetric functions imply that if two or more suppliers choose the same service level, they will receive the same proportion of demand. This type of allocation function is known to be service-maximizing in some special cases when suppliers are identical (Benjaafar et al. 2007, Cachon and Zhang 2007). However, a symmetric function cannot be service-maximizing when suppliers are heterogeneous.

that in our more general case of asymmetric allocation functions and heterogeneous suppliers, a unique Nash equilibrium will not be possible for all forms of $g_i(s_i)$, i = 1, …, N

The following theorem defines a specific form for $g_i(s_i)$ that guarantees both a service-maximizing allocation and a Nash equilibrium. To simplify notation, we introduce the function $\theta_i(s_i)$ which denotes the fraction of supplier *i*'s revenue that is used to cover his demand-independent service costs, i.e., $\theta_i(s_i) = v_i(s_i) / \delta_i \lambda (p - c_i - k_i)$. This fraction is increasing in s_i, with $\theta_i(s_i^{\max}) = 1$.

Theorem 1. *The proportional allocation function defined in* (9) *is service-maximizing for a given* **δ** *when* $g_i(s_i) = \delta_i \theta_i(s_i)^{1/(1-\delta_i)}, 0 < \delta_i < 1$, i = 1, …, *N. Furthermore, if this allocation function is used in the buyer's problem for a given* **δ**, *then the following properties hold.*

(a) *A Nash equilibrium exists, with the suppliers' service levels and profits given by* $s_i^* = s_i^{\max}(\delta_i \lambda)$ *and* $\pi_i(\mathbf{s}^*) = 0$ *for i* = 1, …, *N.*

(b) *Allocation levels at this equilibrium are given by* $\alpha_i(\mathbf{s}^*) = \delta_i$ *for* $i = 1, ..., N$.

(c) *This Nash equilibrium is unique if providing a strictly positive service level* (*i.e.,* $s_i > 0$ for $i = 1, ..., N$) *is a participation condition.*

The specific form of $g_i(s_i)$ presented in theorem 1 provides each supplier with the proper incentive to choose a service level s_i^* which results in $\theta_i(s_i^*) = 1$. This allows the buyer to extract all the channel profit, leaving the suppliers to only cover their costs[8]. The implications of theorem 1 are rather remarkable. By simply manipulating the parameters of the allocation function, the buyer can orchestrate the competition so that each supplier, regardless of his cost structure, has the incentive to spend all his revenue to provide the maximum feasible service level. In addition, the allocation function $\alpha_i(\mathbf{s}^*)$ results in the predefined demand share δ_i at the Nash equilibrium.

<u>Step 2: Characterizing the Optimal Competition Mechanism</u>

We now turn to the more general problem where **δ** is no longer a given set of demand shares, but rather a decision variable for the buyer. Since, for a given vector **δ**, the buyer's profit is increasing in all s_i, $i = 1, ..., N$, it is always optimal for the buyer to use a service maximizing allocation function. Using a service maximizing allocation function, the buyer can then induce equilibrium service levels $s_i^{\max}(\delta_i)$ which zero-out the suppliers' profit. That is,

[8] Unlike previous research involving symmetric allocation functions, it is worth noticing that theorem 1 does not require convexity of the cost functions. Ensuring that $s_i^{\max}(\delta_i \lambda)$ is finite, and thus a finite Nash equilibrium exists, only requires that $\sup_{x>0} v_i(x)$ is strictly greater than $(p - c_i - k_i)\lambda$. This is a reasonable assumption since otherwise the supplier's service cost would never exceed his potential revenue and the supplier would increase his service level unlimitedly.

$$\pi_i^S\left(s_i^{\max}(\delta_i)\right)=\delta_i\lambda\left[p-c_i-f_i\left(s_i^{\max}(\delta_i),\delta_i\right)/\lambda\delta_i\right]=0.$$

Therefore, when the buyer is using a service maximizing allocation function to induce the targeted set of demand shares $\boldsymbol{\delta}$, we can rewrite the buyer's profit function as

$$\pi^B(\mathbf{s}^{\max}(\boldsymbol{\delta}),\boldsymbol{\delta})=(p_B-p)\lambda+\lambda\sum_{i=1}^N\delta_i h\left(s_i^{\max}(\delta_i)\right). \quad (10)$$

This reduces the buyer's problem to an optimization problem with a single vector of decision variables, as defined in the following result.

Theorem 2: *The buyer's problem under the competition mechanism* (5)-(8) *can be solved by using a service maximizing allocation function along with an optimal demand allocation vector,* $\boldsymbol{\delta}^*=(\delta_1^*,...,\delta_N^*)$, *where* $\boldsymbol{\delta}^*$ *is a solution to the following problem*

$$\max_{\boldsymbol{\delta}}\pi^B(\mathbf{s}^{\max}(\boldsymbol{\delta}),\boldsymbol{\delta})=(p_B-p)\lambda+\lambda\sum_{i=1}^N\delta_i h\left(s_i^{\max}(\delta_i)\right), \quad (11)$$

subject to $\delta_i\lambda\le\omega_i$, $i=1,...,N$ *, where* $\mathbf{s}^{\max}(\boldsymbol{\delta})=\left(s_1^{\max}(\delta_1),...,s_N^{\max}(\delta_N)\right)$.

While the theorem does not provide a closed form solution for $\boldsymbol{\delta}^*$, it guarantees the existence of a finite solution which can be computed numerically using a standard multivariable optimization algorithm.

4. Structure of the Optimal Solution

To provide more insight into the structure of the optimal set of demand allocations, $\boldsymbol{\delta}^*$, we first introduce an efficiency measure that can help the buyer evaluate the relative desirability of each supplier. Let e_i denote the supplier's efficiency level, defined as the maximum profit per unit demand that the supplier can generate for the buyer when supplier *i* is given his highest feasible demand allocation. This highest feasible demand allocation, given by $\bar{\delta}_i=\min(\omega_i/\lambda,1)$, depends on the supplier's capacity and the total demand available. The efficiency of supplier *i* is then

$$e_i=(p_B-p)+h\left(s_i^{\max}(\bar{\delta}_i)\right). \quad (12)$$

Without loss of generality, for the remainder of the paper we rename the suppliers in descending order of their efficiencies such that $e_1\ge e_2\ge ...\ge e_N$.

Since this efficiency measure helps order the suppliers by their maximum possible unit contribution to the buyer's profit, a natural heuristic for determining the optimal set of target demand shares, $\boldsymbol{\delta}^*$, would be to allocate as much demand as possible to the most efficient supplier, then allocate as much remaining demand as possible to the next most efficient supplier, and so on until all demand is allocated. As the following result outlines, this solution approach is optimal when the buyer's profit function is convex and the suppliers are homogeneous in cost and/or capacity levels.

Theorem 3. *If the buyer's profit function* (10) *is convex in* $\boldsymbol{\delta}$ *and the suppliers cost structures and/or capacity levels are identical, then the optimal set of demand shares that solves problem* (5) *subject to* (6)-(8) *is*

$$\delta_i^* \lambda = \begin{cases} \omega_i & \text{for } i = 1,\dots,\hat{N}-1 \\ \lambda - \sum_{i=1}^{\hat{N}-1} \omega_i & \text{for } i = \hat{N} \\ 0 & \text{for } i > \hat{N} \end{cases} \quad , \ i = 1,\dots,N \ , \tag{13}$$

where $\hat{N}$ *is the smallest integer such that* $\lambda \le \sum_{i=1}^{\hat{N}} \omega_i$.

Since suppliers have limited capacities, the optimal solution in theorem 3 is to allocate demand to $\hat{N}$ suppliers. More specifically, the buyer should assign full capacity to the first $\hat{N}-1$ most efficient suppliers and then assign any remaining demand to the $\hat{N}^{th}$ most efficient supplier.[9]

When suppliers have identical cost structures, but vary in their capacity levels, the larger suppliers have an advantage in that they can generate more revenue to cover their demand independent service cost, i.e., $v_i(s_i) = v(s_i)$, $i = 1,\dots,N$. In this case, it is easy to show that ordering suppliers by decreasing efficiency levels is equivalent to ordering them by descending capacities. The largest suppliers will be chosen first and given as much demand as they have capacity to fill. On the other hand, when suppliers have identical capacity levels ($\omega_i = \omega$) but their cost structures vary, the efficiency ordering will be in descending order of $s_i^{\max}(\frac{\omega}{\lambda})$, $i = 1,\dots,N$.

To illustrate, consider the following stylized example using simple functions for $v_i(s_i)$ and $h(s_i)$, $i = 1,\dots,N$. In particular, suppose we have nine suppliers which vary in the parameters of their demand dependent cost, k_i, and demand independent cost, $v_i(s_i) = b_i s_i$, with $k_i \in \{2,10,18\}$ and $b_i \in \{40,70,100\}$ for a total of nine combinations as outlined in Table 1(a). Also, suppose $h(s_i) = t\sqrt{s_i}$, t = 10, c = 0, p_B = 100, and total demand is set at $\lambda = 60$. It is easy to verify that the buyer's profit function is convex with respect to $\boldsymbol{\delta}$ for this example, so the conditions of theorem 3 hold.

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[9] For the special case when the capacity of the most efficient supplier is more than the buyer's entire demand, the optimization problem (11) requires allocating the entire demand to the most efficient supplier. Since our competition scheme requires allocating demand to more than one supplier, the buyer can allocate almost the entire demand to the most efficient supplier except for a small fraction which should be allocated to the second efficient supplier. This solution can be arbitrary close to the optimal solution.

Tables 1(b) and 1(c) outline the results of two cases with different capacity configurations. In case I, all suppliers have identical capacities of ω = 10 units, so the efficiency measure reflects differences in costs only. Allocating demand in decreasing order of efficiency leads to optimal demand allocation assignments of 10 units for the first six suppliers (A, D, B, G, E, H) and 0 units for the rest of the suppliers (C, F, I). In case II we increase the capacity of suppliers F, G, H, and I, which increases their efficiency rating and results in a new ordering across suppliers. Now the first two most efficient suppliers (G and A) receive full allocation (30 and 10 units) and the third supplier (H) receives partial allocation (20 units), while no demand is allocated to the rest of the suppliers. In both cases, the efficiency measure makes it easy to determine the optimal demand allocation assignment when comparing across a variety of attributes (i.e., cost structures and capacities).

(a)

			Case I		Case II	
Supplier Name	k	b	ω	efficiency	ω	efficiency
A	2	40	10	101	10	101
B	10	40	10	96	10	96
C	18	40	10	87	10	87
D	2	70	10	96	10	96
E	10	70	10	92	10	92
F	18	70	**10**	85	**30**	89
G	2	100	**10**	93	**30**	103
H	10	100	**10**	90	**30**	97
I	18	100	**10**	84	**30**	88

(b)

Case I Reordered			
Supplier Name	$\delta\lambda$	service level	π^B
A	10	4.5	1,012
D	10	2.6	960
B	10	2.5	958
G	10	1.8	934
E	10	1.4	920
H	10	1.0	900
C	0	0.0	0
F	0	0.0	0
I	0	0.0	0
Total Profit =			5,684

(c)

Case II Reordered			
Supplier Name	$\delta\lambda$	service level	π^B
G	30	5.4	3,097
A	10	4.5	1,012
H	20	2.0	1,946
D	0	0.0	0
B	0	0.0	0
E	0	0.0	0
F	0	0.0	0
I	0	0.0	0
C	0	0.0	0
Total Profit =			6,056

Table 1 – Use of efficiency measures: two examples

While the solution approach outlined in theorem 3 is quite intuitive, it does not guarantee an optimal solution when the suppliers are heterogeneous in both their cost structure and capacities or when the buyer's profit function is non-convex. For example, suppose we have two identical suppliers with the same cost and revenue functions as the previous example. In particular, $v_i(s_i) = 70s_i$ and $h(s_i) = 10\sqrt{s_i}$,

$p_B = 100$ and $\omega_1 = \omega_2 = 59.5$. As mentioned earlier, it is easy to verify that the buyer's profit function is convex with respect to the demand allocations. Figure 1(a) illustrates this result. As we can see, the result of theorem 3 holds and the maximum profit is attained if either supplier receives full allocation ($\lambda\delta_1 = \omega_1 = 59.5$ and $\lambda\delta_2 = \lambda - \omega_1 = 0.05$, or vice versa). However, if we use a more concave revenue function such as $h(s) = 10\sqrt{s} - 4/s^2$, the buyer's profit function is no longer convex and the allocation scheme of theorem 3 no longer provides an optimal allocation assignment. Figure 1(b) illustrates the associated profit function. Here, a partial allocation of $\lambda\delta_1 = 54.6 < \omega_1$ and $\lambda\delta_2 = 0.09 > \lambda - \omega_1$ provides a higher profit for the buyer.

Mathematically, when the objective function of our optimization problem is convex with respect to the decision variables, the maximum point happens on the boundaries of a convex domain. That is, each supplier either receives full allocation or no allocation, except for the supplier which receives the leftover demand to satisfy $\sum_{i=1}^{N} \delta_i = 1$. When the objective function is not convex, it is not possible to derive general analytical solutions. However, as our example shows, the maximum may occur in an interior point, which means a more efficient supplier may receive an allocation lower than his full capacity.

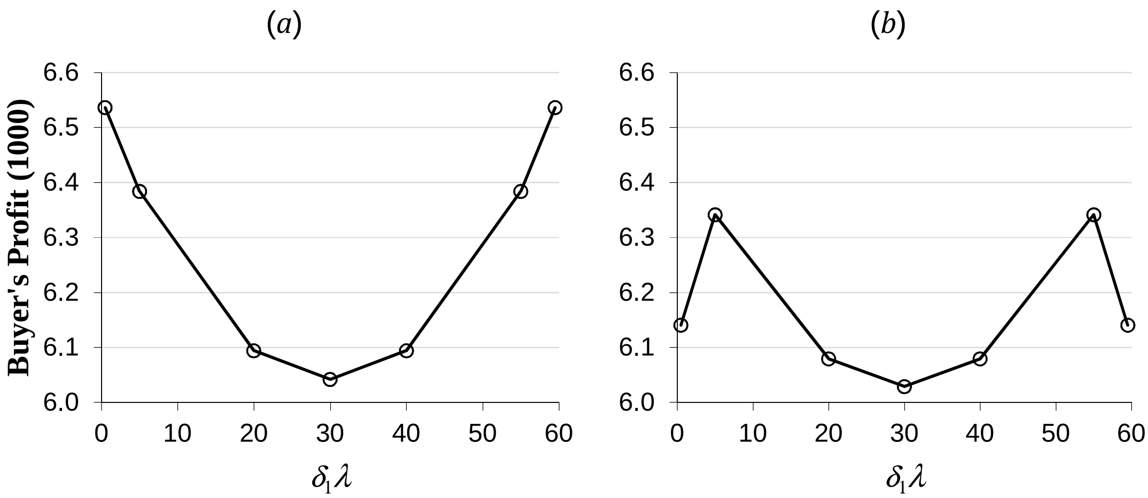

Figure 1 – Impact of revenue function on the optimal allocation

5. Joint Optimization of Demand Allocation and Procurement Prices

We have so far assumed that procurement prices are set exogenously. In this section, we consider the case where the buyer has the power to choose prices optimally and to differentiate these prices across suppliers. In other words, we assume the buyer has the ability to jointly choose the price and allocation vectors. This may arise in settings where the buyer is a dominant player in the supply chain or when suppliers are captive to the demand from a single buyer. This more general case allows us also to examine how procurement prices affect buyer profits and the extent to which differentiated pricing matters.

Let $\mathbf{p} = (p_1, \ldots, p_N)$ denote the vector of procurement prices chosen by the buyer, where p_i is the procurement price from supplier i. The joint pricing and demand allocation problem faced by the buyer can then be stated as

$$\max_{\substack{(\alpha_1(\mathbf{s}),\ldots,\alpha_N(\mathbf{s}))\in\mathscr{F}^N \\ (p_1,\ldots,p_N)\in\mathbf{R}^{N+}}} \pi^B(\mathbf{s},\boldsymbol{\alpha}(\mathbf{s}),\mathbf{p}) = r(\mathbf{s},\boldsymbol{\alpha}(\mathbf{s})) - \sum_{i=1}^{N}(\alpha_i(\mathbf{s})\lambda p_i), \tag{14}$$

subject to:

$$s_i = \arg\max_x \left(\alpha_i(x, s_{-i})\lambda[p_i - c_i - k_i] - v_i(x)\right),\ i = 1,\ldots,N \tag{15}$$

$$\pi_i^S(s_i, s_{-i}) = \alpha_i(s_i, s_{-i})\lambda\left[p_i - c_i - k_i\right] - v_i(s_i) \geq 0,\ i = 1,\ldots,N, \tag{16}$$

$$\alpha_i(\mathbf{s})\lambda \leq \omega_i,\ i = 1,\ldots,N. \tag{17}$$

For a given vector of procurement prices, $\mathbf{p}$, and demand allocations, $\boldsymbol{\delta}$, the buyer's profit is increasing in each of the service levels s_i, $i = 1,\ldots,N$. Consequently, it is optimal again for the buyer to use a service-maximizing allocation function. Let $s_i^{\max}(\delta_i, p_i)$ denote the maximum feasible service level supplier i can provide given demand allocation δ_i and price p_i. Then, we have

$$\pi_i^S\left(s_i^{\max}(\delta_i, p_i)\right) = \delta_i\lambda\left[p_i - c_i - f_i\left(s_i^{\max}(\delta_i, p_i), \delta_i\right)/\lambda\delta_i\right] = 0, \tag{18}$$

or equivalently

$$p_i = c_i + f_i\left(s_i^{\max}(\delta_i, p_i), \delta_i\right)/\lambda\delta_i, \tag{19}$$

for $i = 1, \ldots, N$. Hence, when the buyer uses a service maximizing allocation function to achieve demand allocation vector $\boldsymbol{\delta}$, the buyer's profit function can be rewritten as

$$\pi^B(\boldsymbol{\delta},\mathbf{p}) = \lambda\sum_{i=1}^{N}\delta_i\left[h\left(s_i^{\max}(\delta_i, p_i)\right) - \left[c_i + f_i\left(s_i^{\max}(\delta_i, p_i), \delta_i\right)\right]/\lambda\delta_i\right]. \tag{20}$$

As we can see, the buyer's profit in (20) depends on $\mathbf{p}$ only through $s_i^{\max}(\delta_i, p_i)$. Thus, we can set the value of $s_i^{\max}(\delta_i, p_i)$ to a value that maximizes the buyer's profit (for a given δ_i) by choosing an appropriate value for p_i. We denote by $\sigma_i(\delta_i)$ the value of $s_i^{\max}(\delta_i, p_i)$ which maximizes the buyer's profit. That is,

$$\sigma_i(\delta_i) = \begin{cases} \arg\max_x \left[h(x) - (c_i + f_i(x,\delta_i)/\delta_i\lambda)\right] \text{ if } \delta_i > 0 \\ 0, \text{ otherwise.} \end{cases} \tag{21}$$

Theorem 4 characterizes the optimal competition mechanism.

Theorem 4. *The buyer's problem* (14)-(17) *can be solved by using a service maximizing allocation function along with optimal price vector,* $\mathbf{p}^* = (p_1^*,...,p_N^*)$*, and demand allocation vector,* $\boldsymbol{\delta}^* = (\delta_1^*,...,\delta_N^*)$*, where* $\boldsymbol{\delta}^*$ *is the solution to the following problem*

$$\max_{\boldsymbol{\delta}} \pi^B(\boldsymbol{\delta}) = \lambda \sum_{i=1}^N \delta_i \left[h(\sigma_i(\delta_i)) - (c_i + f_i(\sigma_i(\delta_i),\delta_i)/\lambda\delta_i) \right], \tag{22}$$

subject to $\sum_{i=1}^n \delta_i = 1$ *and* $\delta_i \lambda \le \omega_i$, $i = 1,...,N$. *The optimal vectors of service levels and prices can then be obtained as*

$$s_i^* = \sigma_i(\delta_i^*), \text{ and } p_i^* = c_i + f_i(s_i^*,\delta_i^*)/\lambda\delta_i^*, \; i = 1,...,N.$$

The competition mechanism, as specified by the above solution in theorem 4, has the added feature that it is efficient in the sense that it yields the maximum possible profit for the entire supply chain. In other words, it maximizes the sum of the buyer and suppliers profits.

Corollary 1. *The solution to the buyer's problem* (14)-(17)*, as specified in Theorem 4, maximizes total profit for the supply chain and extracts all the profit for the buyer.*

This result follows from the fact that the optimization problem in (22) does not depend on procurement prices and that the service levels, $\sigma_i(\delta_i)$, are the service levels which maximize buyer's profit from each supplier for the allocated demand, δ_i. The buyer's optimization problem is thus the same as that of a centralized decision maker who makes demand allocations decisions with the objective of maximizing total supply chain profit. The price offered to each supplier is then set equal to the cost per unit of demand allocated to the supplier. An important implication of corollary 1 is that there is no procurement mechanism that can perform better for the buyer than the one specified in theorem 4. The profit realized under this mechanism represents an upper bound over that achieved under any other mechanism. The actual structure of the optimal demand allocation, $\boldsymbol{\delta}^*$, when the buyer can set both the procurement prices and allocation mechanism is similar to that shown in section 4, where procurement prices were fixed. In particular, theorem 3 is still applicable but with supplier efficiency now defined as

$$e_i = p_B + h(\sigma_i(\bar{\delta}_i)) - (c_i + f_i(\sigma_i(\bar{\delta}_i),\bar{\delta}_i)/\lambda\bar{\delta}_i).$$

Since the procurement prices are now endogenous parameters, we cannot define the supplier efficiencies in terms of them, as we did in (12) when procurement prices were exogenous. In the new definition of supplier efficiency, procurement prices are replaced by their values from equation (19). Therefore, the efficiency of each supplier is now characterized in terms of the service level which maximizes the buyer's profit, as indicated in (21).

6. Numerical Results

In this section, we explore the value of using our optimal competition mechanism relative to two competition mechanisms that do not allow for differentiated treatment of suppliers, either in terms of demand allocation or pricing. The first mechanism is the widely studied proportional service allocation function, where the fraction of demand a supplier receives, $\alpha_i(s_i, s_{-i})$, is set in proportion to the supplier's relative service level [10]

$$\alpha_i(s_i, s_{-i}) = s_i / \sum_{j=1}^{N} s_j. \tag{23}$$

The second mechanism follows our optimal mechanism structure except that the procurement prices are constrained to be equal for all suppliers. In other words, the buyer chooses a common procurement price, which is the solution to the following optimization problem:

$$\max_{\boldsymbol{\delta}, p} \pi^B(\mathbf{s}^{\max}(\boldsymbol{\delta}), \boldsymbol{\delta}) = (p_B - p)\lambda + \lambda \sum_{i=1}^{N} \delta_i h\left(s_i^{\max}(\delta_i)\right), \tag{24}$$

subject to the constraints stated in Theorem 2.

The numerical results we present are for a specific application which is an extension of an example originally introduced in Benjaafar et al. (2007). This extension allows suppliers to now be heterogeneous in cost and capacity. Although our numerical results are specific to this example application, the general insights apply more broadly to other applications that fit our model assumptions.

Example description: Consider a system with one buyer and *N* suppliers. Demand is in the form of orders that occur continuously over time according to a Poisson process with rate λ. Orders are forwarded to each supplier based on the fraction of demand the supplier is allocated, such that if supplier *i* receives fraction δ_i, then the rate of orders it receives is $\delta_i\lambda_i$. Suppliers process orders one at a time, with processing time at supplier *i* being exponential with rate μ_i. Each supplier's service level is defined by his probability of meeting a given lead-time target τ, i.e., $s_i = \Pr(W_i \le \tau)$ where W_i is a random variable representing lead-time for supplier *i*. Suppliers have the ability to increase their service levels by

[10] The proportional allocation can be used in a setting with capacitated suppliers by modifying the supplier profit function such that an allocation beyond a supplier's capacity leads to no additional revenue for that supplier.

investing in processing capacity (processing rates) at an amortized cost of k_i per unit of capacity. In addition, supplier i incurs production cost c_i for each item produced. The above implies that the suppliers' service cost functions are given by

$$f_i(s_i,\delta_i) = k_i\lambda\delta_i + v_i(s_i) = k_i\lambda\delta_i - k_i \ln[1/(1-s_i)]/\tau, \quad i = 1,\ldots,N,$$

where $s_i = \Pr(W_i \le \tau) = 1 - e^{-(\mu_i-\lambda_i)\tau}$, see Benjaafar et al. (2007) for further details. The buyer's revenue function is given by the following relatively general quadratic concave form:

$$r(\mathbf{s},\boldsymbol{\delta}) = p_B\lambda - \sum_{i=1}^{N}\delta_i\lambda t\left(1-s_i\right)^2,$$

where t is a positive constant.

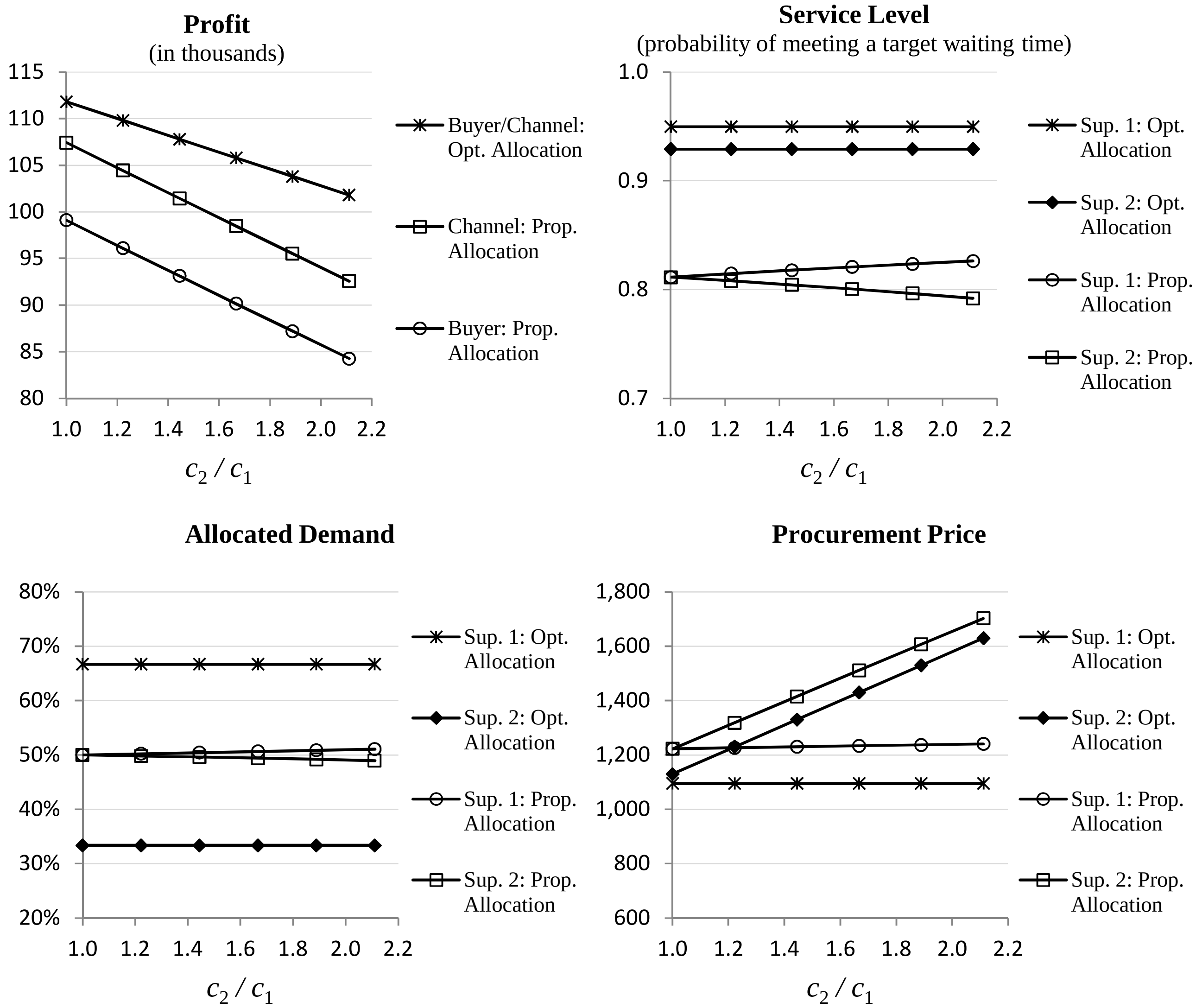

Figure 2 – Impact of optimal allocation function
$N = 2;\ \ \lambda = 60;\ \ \tau = 1;\ \ p_B = 2980;\ \ t = 2980;\ \ k_1 = k_2 = 600;\ \ \omega_1 = \omega_2 = 40;\ \ c_1 = 450;$

Figure 2 compares the performance of our optimal competition mechanism against the service proportional allocation mechanism, defined by (23), for a system with two suppliers. Note that price is endogenous and differentiated under both mechanisms. We see that the benefit of using an optimal allocation can be substantial and increases with the difference in the production costs of the suppliers. The optimal allocation leads to both substantially higher service levels and channel profits. Notice that under the service proportional allocation, suppliers do not provide the maximum feasible service level, and therefore some of the channel profit stays with the suppliers. This explains the gap between buyer profit and channel profit. Surprisingly, even though service levels are substantially lower under the service proportional allocation, the buyer pays higher prices than under the optimal allocation (without these higher prices, service levels would be even lower). Notice also that service levels and demand allocations are unaffected by differences in supplier costs under the optimal allocation. This is because demand allocations (and corresponding service levels) are only determined by the relative ranking of the costs and not their specific values.

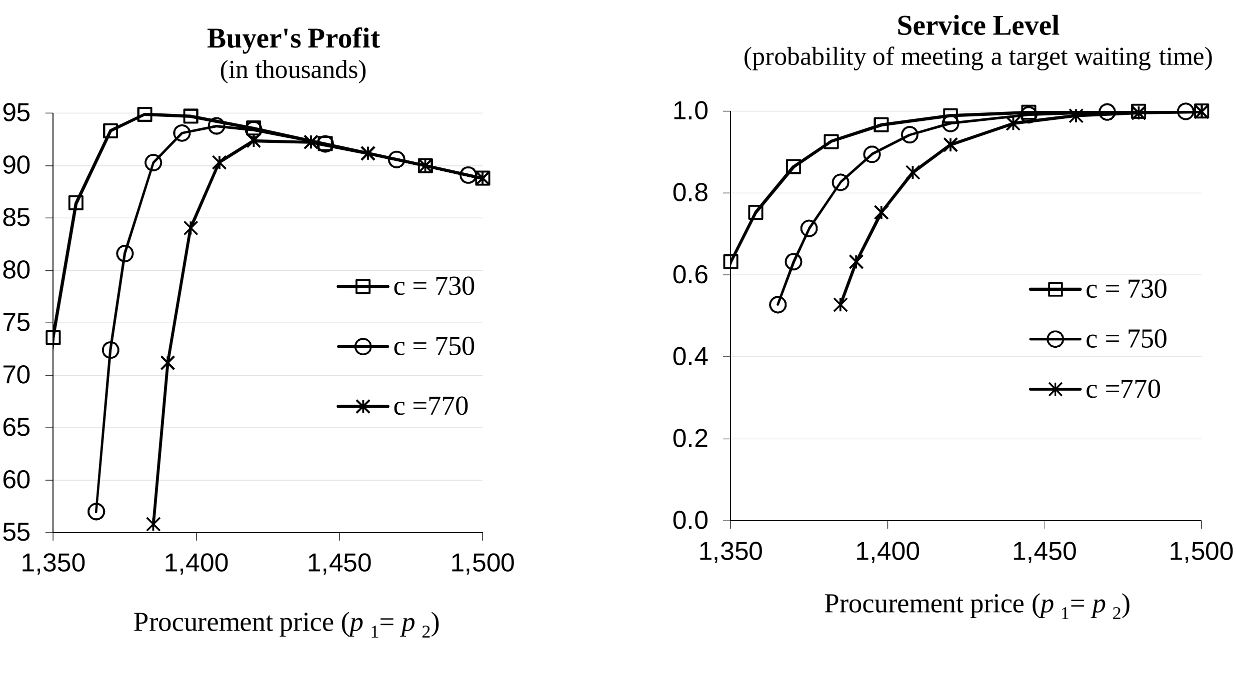

Figure 3 – Impact of procurement price on buyer's profit and service level

Figure 3 illustrates the impact of procurement prices under the optimal allocation when prices are exogenously set and common among the suppliers, i.e. under the assumptions of Section 4[11].

As we can see, profit is quite sensitive to price, with profit initially increasing in price and then decreasing. Service level is increasing in price, but at a diminishing rate. These results highlight the importance of choosing prices optimally even if they cannot be differentiated.

Figures 4 and 5 compare the performance of the optimal competition mechanism described in Section 5, where prices can be differentiated, with the competition defined by (24) where the buyer is constrained to set a common procurement price. As we can see, differentiated pricing can have a significant impact on buyer profit, with the difference in profit increasing in the differences in cost and capacities. Note that with differentiated pricing, the prices paid to the suppliers can vary substantially, with this difference affected by both production costs and capacities. It is also interesting to observe how asymmetry in capacity affects buyer's profit with differentiated versus common pricing. In particular, higher capacity asymmetry is beneficial to the buyer when prices can be differentiated (e.g., when the low cost supplier has more capacity than the high cost supplier, the buyer benefits by shifting more demand to the low cost supplier and paying that supplier a lower price). In contrast, higher capacity asymmetry is harmful to the buyer when prices cannot be differentiated (e.g., when the low cost supplier has more capacity than the high cost supplier, the buyer shifts more demand to the low cost supplier but now must pay a higher price to make up for the lower demand allocated to the high cost supplier).

7. Concluding Remarks

In this paper, we showed how to design an optimal supplier competition when suppliers are heterogeneous in their cost structures and capacity levels and the buyer's revenue is dependent on the service levels provided by the selected suppliers and the amount of demand allocated to these suppliers. We showed how a demand allocation function can be designed to induce suppliers to provide the maximum feasible service level regardless of the level of heterogeneity in the cost structures and capacities of the suppliers. In settings where it is desirable to set the demand allocation in a predetermined way, we showed that it is possible to design the allocation function to induce this desired allocation as an outcome of the competition. In settings where the buyer can choose both demand allocation and procurement prices, we showed how the buyer can maximize total profit for the supply chain while

[11] Figure 3 uses the same parameter values as those used in figure 2.

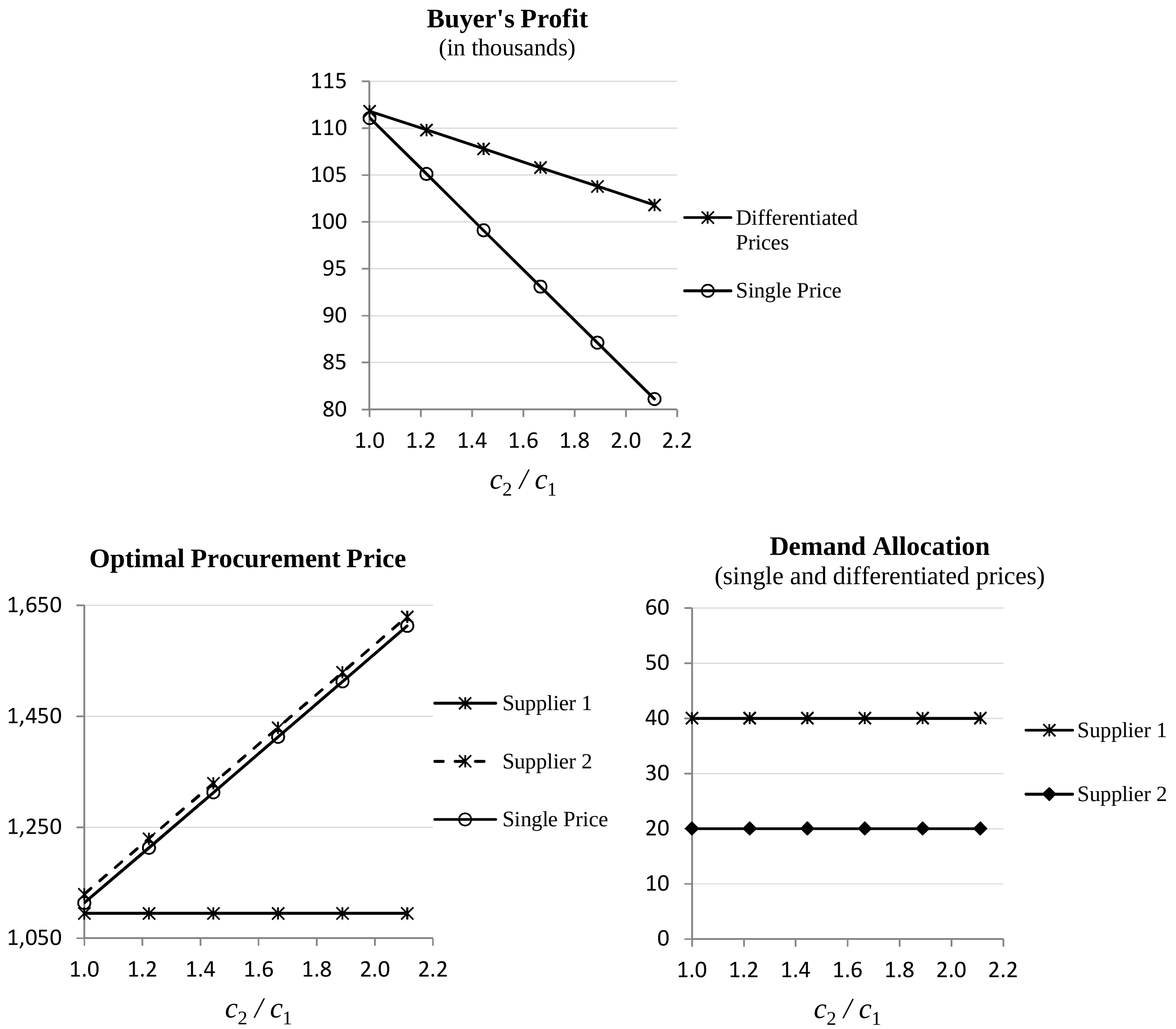

Figure 4 – Single versus differentiated procurement prices: impact of cost heterogeneity
$\omega_1 = \omega_2 = 40$; $c_1 = 450$;

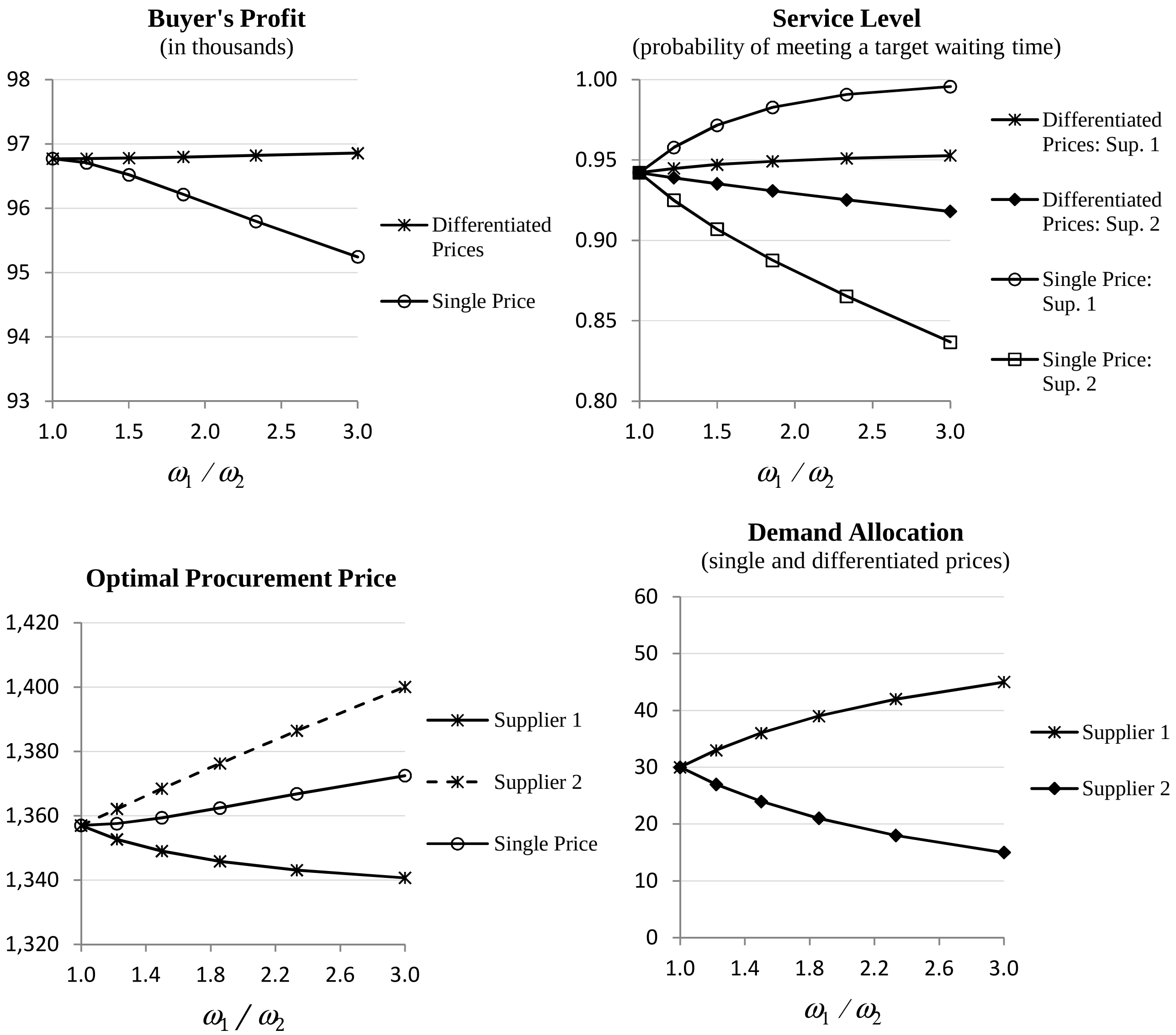

Figure 5 – Single versus differentiated procurement prices: impact of capacity heterogeneity
($c_1 = c_2 = 400; \quad \omega_1 + \omega_2 = 60$)

continuing to extract all the profit for herself. Finally, we showed that the advantage of using an optimal demand allocation and optimal procurement price can be considerable when suppliers are heterogeneous.

There are several possible avenues for future research. We know from the results of this study that the ability to differentiate competition terms is critical when the suppliers are heterogeneous. However, full differentiation of the form implied by our optimal solutions is not always feasible due in some cases due to regulations or supplier resistance. A possible alternative in such cases might be to divide the suppliers into groups and offer the same competition terms within a group (e.g., suppliers located in the same region or country). A group-based strategy raises new questions for how the buyer should design the optimal competition, how the performance of each scheme is affected by varying levels of grouping, and how degrees of similarity or dissimilarity in cost and capacity between groups (and between members of each group) affect buyer and suppliers' profit.

Our analysis currently assumes that the buyer's revenue increases with service quality while the demand rate is fixed. If demand were also an increasing function of service, the suppliers' incentive to invest in service would change since an increase in service could increase the supplier's allocation as well as the overall demand level available to all suppliers. It would be interesting to study how this relationship between customer demand and service, including the possibility of free riding by suppliers, might change the form of the optimal competition design.

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Appendix

Proof of Theorem 1

We first prove part (*c*) of the theorem. Let us write the profit functions of the suppliers in terms of $y_i = g_i(s_i) = \delta_i \theta_i (s_i)^{1/(1-\delta_i)}$. That is,

$$\pi_i(y_i, y_{-i}) = \frac{y_i}{G}\lambda r_i - \lambda r_i \delta_i^{\delta_i} y_i^{(1-\delta_i)} \tag{A1}$$

where $G = \sum_{k=1}^{N} y_k$ and $r_i = p_i - c_i - k_i$. The constraint that all suppliers provide positive service levels is equivalent to $y_i > 0,\ i = 1,\ ...,\ N$. Since $y_i > 0$, a Nash equilibrium point must satisfy the first order optimality condition. That is, in order to prove part (*c*) of the theorem, we need to show that the following system of (*N*+1) equations with unknowns $y_i,\ i = 1,...,N$ and *G* has a unique solution that maximizes (A1).

$$\frac{\partial \pi_i(y_i, y_{-i})}{\partial y_i} = \frac{G - y_i}{G^2}\lambda r_i - \lambda r_i \delta_i^{\delta_i}(1-\delta_i) y_i^{-\delta_i} = 0, \quad i = 1,...,N, \quad \text{and} \quad G = \sum_{k=1}^{N} y_k, \tag{A2}$$

or, equivalently,

$$Y_i^{\delta_i}(1-Y_i) - \delta_i^{\delta_i}(1-\delta_i)G^{1-\delta_i} = 0, \quad i = 1,...,N, \text{ and} \tag{A3}$$

$$\sum_{k=1}^{N} Y_k = 1, \tag{A4}$$

where $Y_i = y_i / G$. We can rewrite the first order optimality conditions as

$$D(Y_i) = \delta_i^{\delta_i}(1-\delta_i)G^{1-\delta_i},\ i = 1,\ ...,\ N,\ \sum_{k=1}^{N} Y_k = 1, \tag{A5}$$

where $D(Y_i) \equiv Y_i^{\delta_i}(1-Y_i)$. We can see that $D(0) = D(1) = 0$, $D(Y_i) > 0$ for $0 < Y_i < 1$, and $D(Y_i) < 0$ for $Y_i > 1$. Also, $D(Y_i)$ has a maximum at $\delta_i/(1+\delta_i)$ which is equal to $\delta_i^{\delta_i}/(1+\delta_i)^{1+\delta_i}$. Hence, for any $G < G^{\max} = \left(1/(1-\delta_i)(1+\delta_i)^{1+\delta_i}\right)^{1/1-\delta_i}$, equation (A5) has two solutions $0 < Y_{i,1} < Y_{i,2} < 1$. We want to argue that $Y_{i,1}$ corresponds to a local minimum for supplier *i*'s profit function, (A1). We observe that the sign of the derivative of the profit function of supplier *i*, equation (A3), changes from negative to positive when we increase Y_i from values smaller than $Y_{i,1}$ to values bigger than $Y_{i,1}$. We also observe that, for fixed decisions of the other suppliers, $Y_i = y_i/(y_i + \sum_{j\neq i} y_j)$ is increasing in y_i. Therefore, when we increase y_i, the sign of equation (A3) changes from negative to positive at $y_i = \left(Y_{i,1}/(1-Y_{i,1})\right)\sum_{j\neq i} y_j$, which in turn means that $Y_{i,1}$ corresponds to a local minimum of supplier *i*'s profit function. Thus, $Y_{i,1}$ cannot be a feasible solution. Similarly, we can show that $Y_{i,2}$ corresponds to a local maximum of supplier *i*'s profit function. Therefore, $Y_{i,2},\ i=1,...,N$ is the unique solution to the system of equations (A5) which corresponds to the values of y_i that maximize the profit functions of the suppliers. Figure A1 graphically illustrates the above argument.

Lemma A1 below shows that $G^{\max} > 1$ for any $0 < \delta_i < 1$. For $G = 1$, the only solution for equations in (A5) that maximizes profit function (A1) is $Y_{i,2} = \delta_i$. It is easy to see that $Y_{i,2}$ is decreasing in *G* (see figure A1). Therefore, for $G < 1$, we have $Y_{i,2} > \delta_i$ or equivalently $\sum_{i=1}^{N} Y_{i,2} > 1$, which is not a feasible solution. Also, for $G > 1$, we have $Y_{i,2} < \delta_i$ or equivalently $\sum_{i=1}^{N} Y_{i,2} < 1$, which is not a feasible solution as well. Therefore, $Y_i = \delta_i$ and $G = 1$ (or equivalently $y_i^* = \delta_i$) is the unique solution of the system of equations (A3)-(A4) which maximizes each profit function in (A1), given all suppliers provide a positive service level.

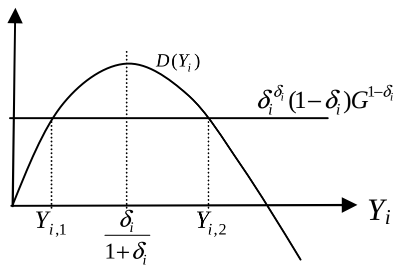


Figure A1 – The solution to first order optimality condition (A5)

We now prove part (*a*) and (*b*) of the theorem. We showed that $y_i^* = \delta_i$ satisfies the first order optimality condition (A2). If we release the positive service level constraint, $y_i^* = \delta_i,\ i=1,...,N$ is still a Nash equilibrium since any supplier *j* cannot increase his profit by choosing $y_j = 0$ while other suppliers choose $y_i^* = \delta_i,\ i \neq j$. We can easily verify that $y_i^* = \delta_i$ results in $s_i^* = s_i^{\max}(\delta_i \lambda)$, $\pi_i(\mathbf{s}^*) = 0$, and $\alpha_i(\mathbf{s}^*) = \delta_i$.

Lemma 3: For any $0 < x < 1$, we have $\left(1/(1-x)(1+x)^{1+x}\right)^{1/(1-x)} > 1$.

Proof of Lemma 3: It is enough to prove that $Z(x) = (1-x)(1+x)^{1+x} < 1$ for any $0 < x < 1$. We can see that $Z(0) = 1$ and $Z(1) = 0$. Therefore, it is enough to show that $Z(x)$ is decreasing in the interval $(0, 1)$. The derivative of $Z(x)$ can be written as

$$\frac{dZ(x)}{dx} = (1+x)^{1+x}\left[(1-x)\left(\ln(1+x)+1\right)-1\right] = (1+x)^{1+x}\left[X(x)-1\right],$$

where $X(x) = (1-x)\left(\ln(1+x)+1\right)$. Noticing that $X(0) = 1$, $X(1) = 0$, and

$$\frac{dX(x)}{dx} = -\left(\ln(1+x)+1\right) + \frac{1-x}{1+x} < 0,$$

we can conclude that $X(x)$ is decreasing and less than 1 in $(0,1)$. Hence, $Z(x)$ is decreasing and less than 1 in $(0,1)$. This completes the proof of the lemma.

Proof of Theorem 2

Considering the discussion under step 2 in section 3, the proof is straightforward. The existence of a solution is guaranteed by the fact that a function with bounded values always has a maximum on a compact domain.

Proof of Theorem 3

Here, we present the proof for the case when the suppliers have identical cost structures. The proof for the case of identical suppliers' capacities is omitted since it follows the same type of reasoning.

When $\omega_1 \geq \lambda$, it is easy to show that the optimal allocation is

$$\delta_i^* \lambda = \begin{cases} \omega_i & for\ i = 1 \\ 0 & for\ i > 1 \end{cases},\ i = 1,\ldots,N,$$

For the case of $\omega_1 < \lambda$, since the suppliers have identical cost structures, the difference in suppliers' efficiencies is solely due to difference in their capacities. Therefore, when we rename suppliers according to their descending order of their efficiencies, in fact, it is according to their descending order of their capacities. Hence we have $\overline{\delta}_1 \geq \overline{\delta}_2 \geq \ldots \geq \overline{\delta}_N$. Then we can rewrite (13) as

$$\delta_i^* = \begin{cases} \overline{\delta}_i & for\ i = 1,\ldots,\hat{N}-1 \\ 1-\sum_{i=1}^{\hat{N}-1}\overline{\delta}_i & for\ i = \hat{N} \\ 0 & for\ i > \hat{N} \end{cases},\ i = 1,\ldots,N \tag{A6}$$

Let $\psi(\delta_i) = \delta_i \lambda[h(\sigma(\delta_i)) - c - k] - v(\sigma(\delta_i))$. Hence we have $\tilde{\pi}^B(\boldsymbol{\delta}) \equiv \pi^B(\boldsymbol{\sigma}(\boldsymbol{\delta}), \boldsymbol{\delta}) = \sum_{i=1}^{N} \psi(\delta_i)$. Notice that $\tilde{\pi}^B(\boldsymbol{\delta})$ is convex in $\boldsymbol{\delta}$ if and only if $\psi(\delta_i)$ is convex in δ_i. Here, we propose a procedure through which we can build $\boldsymbol{\delta}^* = (\delta_1^*, ..., \delta_N^*)$ from any feasible initial set of $\boldsymbol{\delta} = (\delta_1, ..., \delta_N)$. We then show that we always have $\tilde{\pi}^B(\boldsymbol{\delta}^*) \geq \tilde{\pi}^B(\boldsymbol{\delta})$, which proofs the proposition. At each round of this procedure, we modify only two elements of allocation vector until we achieve $\boldsymbol{\delta}^*$.

Step 1: $m = 0; \quad i = 1; \quad j = N;$

Step 2: $\delta_t^m = \delta_t \quad for \quad t = 1, ..., N$

Step 3: $\Delta = \min(\delta_j^m, \overline{\delta}_i - \delta_i^m)$

Step 4: $\delta_i^{m+1} = \delta_i^m + \Delta; \quad \delta_j^{m+1} = \delta_j^m - \Delta; \quad \delta_t^{m+1} = \delta_t^m \quad for \quad t \neq i, j$

Note: By virtue of Lemma 2 below, we have $\tilde{\pi}^B(\boldsymbol{\delta}^{m+1}) \geq \tilde{\pi}^B(\boldsymbol{\delta}^m), \; where \; \boldsymbol{\delta}^m = (\delta_1^m, ..., \delta_N^m).$

Step 5: $if \quad \delta_i^{m+1} = \overline{\delta}_i \quad then \quad i = i + 1$

Step 6: $if \quad \delta_j^{m+1} = 0 \quad then \quad j = j - 1;$

Step 7: $if \quad i < j \quad then \quad m = m + 1 \quad and \quad return \; to \; step \; 3$

$else \quad \delta_t^* = \delta_t^{m+1} \quad for \quad t = 1, ..., N; \quad end \; of \; procedure;$

In any round of this procedure we remove all or part of the allocation from a low capacity supplier and add the same amount of allocation to a higher capacity supplier. By virtue of Lemma 2, this reallocation results in higher centralized profit. We repeat this procedure until we cannot reallocate demand from a lower capacity supplier to a higher capacity supplier. It is not difficult to see that at the end of this procedure we have the allocation stated in (13). Since $\tilde{\pi}^B(\boldsymbol{\delta}^*) \geq \tilde{\pi}^B(\boldsymbol{\delta})$ for any $\boldsymbol{\delta}$, we can conclude that $\boldsymbol{\delta}^*$ is the optimal allocation. This concludes the proposition.

Lemma 2: If $\overline{\delta}_i \geq \overline{\delta}_j$ then $\psi(\delta_i) + \psi(\delta_j) \leq \psi(\delta_i + \Delta) + \psi(\delta_j - \Delta)$ where $\Delta = \min(\delta_j^k, \overline{\delta}_i - \delta_i^k)$.

<u>Proof of Lemma 2.</u>

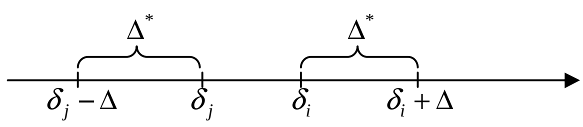
From the definition of Δ we have:

$$\delta_i + \Delta = \min(\overline{\delta}_j + \delta_i, \overline{\delta}_j) \text{ and } \delta_j - \Delta = \max(0, \delta_j - (\overline{\delta}_i - \delta_i)).$$

Since $\overline{\delta}_i \geq \overline{\delta}_j \geq \delta_j \geq 0$ we have $\delta_i + \Delta = \delta_j$ and $\delta_j - \Delta = \delta_i$.

To prove this lemma we consider the following two possible cases:

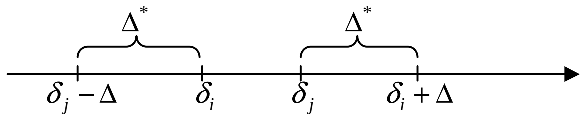
(1) $\delta_i \geq \delta_j$



$$\Delta^* = (\delta_i + \Delta) - \delta_i = \delta_j - (\delta_j - \Delta) = \Delta$$

In this case let $d_1 = \delta_j - \Delta$; $d_2 = \delta_j$; $d_3 = \delta_i$; $d_4 = \delta_i + \Delta$.

(2) $\delta_i < \delta_j$



$$\Delta^* = (\delta_i + \Delta) - \delta_j = \delta_i - (\delta_j - \Delta)$$

In this case let $d_1 = \delta_j - \Delta$; $d_2 = \delta_i$; $d_3 = \delta_j$; $d_4 = \delta_i + \Delta$.

Since $\psi(.)$ is a convex function, we have $\psi'(d_1) \le \psi'(d_2) \le \psi'(d_3) \le \psi'(d_4)$ where $\psi'(x) = \partial \psi(x) / \partial x$.

Again from the convexity of $\psi(.)$ we can conclude that

$$\frac{\psi(d_2) - \psi(d_1)}{\Delta^*} \le \psi'(d_2) \le \psi'(d_3) \le \frac{\psi(d_4) - \psi(d_3)}{\Delta^*}$$

Therefore,

$$\psi(d_2) - \psi(d_1) \le \psi(d_4) - \psi(d_3) \quad \Rightarrow \quad \psi(d_2) + \psi(d_3) \le \psi(d_1) + \psi(d_4) \qquad \text{(A11)}$$

It is easy to verify that, for both cases (1) and (2), the inequality in (A11) is equivalent to

$$\psi(\delta_i) + \psi(\delta_j) \le \psi(\delta_i + \Delta) + \psi(\delta_j - \Delta)$$

This concludes the proof of the lemma.

<u>Proof of Theorem 4</u>

Considering the discussion before theorem 4, the proof is straightforward.