Supplementary FIG. 1. **Noise dependence of the order parameter dynamics.** Panel a: Average over the ring of condensate density as a function of time for two different values of the noise amplitude differing by a factor of ten, shown in red and black. Panel b shows a blowup of the ‘knee’ region where the condensation process first happens. We see that this process is largely insensitive to the noise amplitude.
Supplementary FIG. 2. **Dynamic susceptibility in the broken phase.** Panel a: Poles in dynamic correlation function at $T/T_c = 0.9$. There is a Goldstone pole at the origin. The leading pole nearest the real axis, corresponding to the amplitude or ‘Higgs’ mode, with imaginary part $\omega_*$ gives the equilibration time. Panel b: inverse of the imaginary part of leading pole $\omega_*$ as the critical point is approached. This directly gives the equilibration time $\tau$. The best fit result gives $\tau \sim 0.8\epsilon^{-1}$, with a $\tau_0$ differing from the unbroken phase. Again the explicit calculation agrees with the analytical derivation that $z = 2$. The pole structure found here resembles closely the AdS$_4$ results of [1, 2].
Supplementary FIG. 3. **Static susceptibility in broken phase.** Panel a: poles in static correlation function at $T/T_c = 0.9$. The leading poles nearest the real axis, with imaginary part $k_*$ give the correlation length. Panel b: imaginary part of leading pole $k_*$ as the critical point is approached from below. The correlation length $\xi = k_*^{-1}$ diverges as $\xi = \xi_0 \epsilon^{-1/2}$. We determined $\xi_0 = 0.39 \pm 0.01$ from numerically solving Eq. (41).
