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# INVENTORY COMPETITION IN MAKE-TO-STOCK SYSTEMS

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**Abstract:** We present models for competition among multiple suppliers for demand from a single manufacturer. The suppliers produce to stock a single product and are allocated demand by the manufacturer based on the amount of inventory they hold. We prove the existence of a Nash equilibrium for a broad class of market allocation schemes. For the special case of identical suppliers under either a stock-proportional or fill rate-proportional allocation, we show the uniqueness of the Nash equilibrium. Analysis of the Nash equilibrium for this case reveals that (a) the manufacturer benefits from competition (in the form of higher fill rates), (b) the manufacturer benefits more from a stock-proportional allocation than a fill rate-proportional allocation, and (c) the manufacturer benefits the most when the number of suppliers is two.

## 1 Introduction

In many industries, original equipment manufacturers (OEM) are turning to outsourcing as an alternative to in-house manufacturing. This trend is particularly evident in the electronics industry where the contract manufacturing sector is now a \$180 billion industry and expected to represent over 50% of all electronics manufacturing by 2005 (Kador, 2001). One of the decisions that an OEM has to make is how much of their manufacturing they should outsource and to whom they should outsource it. In the electronics industry, there are numerous contract manufacturers with nearly similar capabilities. The issue for an OEM is then whether to outsource to a single supplier or to allocate demand among multiple ones. There is an increasing consensus that multiple sourcing is beneficial to the OEM's by creating redundancy in their supplier lines and encouraging price and service among their suppliers. However, with multiple sourcing, the OEM's must determine how demand should be allocated among the multiple suppliers.

In this paper, we explore inventory-based competition as a mechanism for the manufacturer to allocate demand among a set of potential suppliers. In particular, the manufacturer rewards suppliers that hold more inventory (provide a higher service level) with a higher market share. We assume that the suppliers adjust their capacity proportionally to the amount of market share they are allocated. The suppliers earn revenue per unit supplied to the manufacturer and incur holding and backordering costs. Demand at the manufacturer occurs one unit at a time. The probability that a particular supplier is allocated the order is determined by their market share allocation. The suppliers manage their inventory using a base-stock policy, with these base-stock levels defining the competitive strategy of each supplier.

In this paper, we prove the existence of a Nash equilibrium for a broad class of market allocation schemes. For the special case of identical suppliers under either a stock-proportional or fill rate-proportional allocation, we show the uniqueness of the Nash equilibrium. Analysis of the Nash equilibrium for this case reveals that (a) the manufacturer benefits from competition (in the form of higher fill rates), (b) the manufacturer benefits more from a stock-proportional allocation than a fill rate-proportional allocation, and (c) the manufacturer benefits the most when the number of suppliers is two.

Our work is related to the growing body of research on inventory-based competition. Examples include Parlar (1988), Karjalainen (1992), Lippman and McCardel (1997), Mahajan and Van Ryzin (2001), and Wang and Gerchack (2001). For a recent review and additional references see (Cachon 2003).

## 2 Model Description

There are  $N$  suppliers who produce a single product in a make-to-stock fashion using a base-stock policy. The suppliers compete for a fixed market share from a single manufacturer. Market share is allocated so as to reward suppliers that hold more inventory (market share is non-decreasing in the supplier's base-stock level). The suppliers are free to choose their stocking levels and incur inventory holding and backordering costs. In response to a market share allocation, the suppliers increase their capacity proportionally to maintain a fixed target utilization  $\mathbf{r}_i$ , where  $0 = \mathbf{r}_i = \mathbf{r}_o < 1$  and  $\mathbf{r}_o$  is exogenously set by the manufacturer. Each supplier  $i$  incur a holding cost  $h_i$  per unit of inventory per unit time and a backorder cost  $b_i$  per unit backordered per unit time, a production  $c_i$  per unit produced, and a capacity cost  $k_i$  per unit of capacity (measured in terms of the associated production rate). The supplier's revenue has a fixed component and a variable component. The fixed component,  $g_i$ , is guaranteed revenue for agreeing to be a supplier. The variable component,  $p_i$ , is earned per actual unit supplied. Demand at the manufacturer occurs according to a Poisson process with rate  $\mathbf{I}$ . The fraction of demand allocated to supplier  $i$  is denoted by  $\mathbf{a}_i$ , where  $0 = \mathbf{a}_i = 1$  and  $\mathbf{a}_1 + \mathbf{a}_2 + \dots, \mathbf{a}_N = 1$ . The parameter  $\mathbf{a}_i$  can be viewed as the probability that incoming demand is allocated to supplier  $i$ . Although a truly probabilistic allocation is unlikely in practice, it is useful in approximating the behavior of a central dispatcher that attempts to adhere to a specified market share for each supplier. It is also useful in modeling settings where demand arises from a sufficiently large number of sources. The variable  $\mathbf{a}_i$  corresponds in that case to the fraction of demand sources (e.g., geographical locations) that are always satisfied from supplier  $i$ . Production times at each supplier are exponentially distributed with rate  $\mathbf{m}_i = \mathbf{a}_i \mathbf{I} / \mathbf{r}_i$ . Finished goods at the suppliers are managed according to a base-stock policy with base stock level  $s_i$  ( $s_i = 0$ ) at supplier  $i$ . This means that the arrival of demand at supplier  $i$  always triggers a replenishment order with the supplier's production system. Hence, the production system at each supplier behaves like an M/M/1 queue.

## 3 The Supplier's Problem

For a given market allocation  $\mathbf{a}_i$  and a base level  $s_i$ , the expected profit  $\mathbf{p}_i$  at each supplier  $i$  is given by

$$\mathbf{p}_i(\mathbf{a}_i, s_i) = \mathbf{a}_i \mathbf{I} (p_i - c_i) + g_i - k_i \mathbf{m}_i - h_i E(I_i) - b_i E(B_i); \quad i=1, \dots, N, \quad (1)$$

where  $E(I_i)$  and  $E(B_i)$  denote respectively expected inventory level and expected backorder level at supplier  $i$ . Under our assumptions,  $E(I_i)$  and  $E(B_i)$  are given by (see for example (Buzacott and Shanthikumar, 1993))

$$E(I_i) = s_i - \frac{\mathbf{r}_i}{1 - \mathbf{r}_i} (1 - \mathbf{r}_i^{s_i}) \text{ and } E(B_i) = \frac{\mathbf{r}_i^{s_i+1}}{1 - \mathbf{r}_i}, \quad (2)$$

from which we can rewrite expected profit as

$$\mathbf{p}_i(\mathbf{a}_i, s_i) = \mathbf{a}_i \mathbf{l} r_i + g_i - h_i \left( s_i - \frac{\mathbf{r}_i}{1 - \mathbf{r}_i} \right) - (h_i + b_i) \frac{\mathbf{r}_i^{s_i+1}}{1 - \mathbf{r}_i}. \quad (3)$$

where  $r_i = (p_i - c_i) - k_i / \mathbf{r}_i$ . Clearly,  $g_i$  should satisfy the condition  $g_i \geq b_i \mathbf{r}_i / (1 - \mathbf{r}_i)$  so that  $\mathbf{p}_i = 0$  for all values of  $s_i$ .

For a fixed market allocation  $\mathbf{a}_i$ , the supplier's problem can be stated as

$$\max_{s_i} \mathbf{p}_i(\mathbf{a}_i, s_i); \quad (4)$$

The value of  $s_i, s_i^n$ , that maximizes  $\mathbf{p}_i$ , is given by  $s_i^n = 0$  if  $\mathbf{a}_i = 0$ . Otherwise (if  $\mathbf{a}_i > 0$ ),  $s_i^n$  is given by

$$s_i^n = \begin{cases} \frac{\ln[\mathbf{b}_i \frac{h_i}{b_i + h_i}]}{\ln(\mathbf{r}_i)} & \text{if } \mathbf{b}_i < \frac{h_i + b_i}{h_i} \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

where  $\mathbf{b}_i = -(1 - \mathbf{r}_i) / \mathbf{r}_i \ln(\mathbf{r}_i)$ . The superscript  $n$  is used as mnemonic for no competition. Substituting  $s_i^n$  in the profit function, we obtain

$$\mathbf{p}_i(\mathbf{a}_i, s_i^n) = \mathbf{a}_i \mathbf{l} r_i - h_i \left[ s_i^n - \frac{\mathbf{r}_i}{1 - \mathbf{r}_i} (1 - \mathbf{b}_i) \right] + g_i, \quad (6)$$

for  $i = 1, 2, \dots, N$ .

The value of  $s_i^n$  can be usefully approximated by

$$\tilde{s}_i = \frac{\ln[h_i / (h_i + b_i)]}{\ln(\mathbf{r}_i)}, \quad (7)$$

the solution to the difference equation  $\mathbf{p}_i(\mathbf{a}_i, s_i + 1) - \mathbf{p}_i(\mathbf{a}_i, s_i) = 0$ . It can be shown that  $|s_i^n - \tilde{s}_i| < 1$  and  $s_i^n / \tilde{s}_i \rightarrow 1$  as  $\mathbf{r}_i \rightarrow 1$ , hence asymptotically exact. If the approximation  $s_i^n \approx \tilde{s}_i$  is used, we obtain the simpler expression

$$\mathbf{p}_i(\mathbf{a}_i, s_i^n) \approx \mathbf{a}_i \mathbf{l} r_i - h_i \tilde{s}_i + g_i. \quad (8)$$

#### 4 The Market Allocation

There are a variety of ways in which the manufacturer may allocate market share among the competing suppliers to reward those that hold more inventory. We use the notation  $\mathbf{a}_i(\mathbf{s})$ , where  $\mathbf{s} = (s_1, s_2, \dots, s_N)$  to emphasize the dependence of  $\mathbf{a}_i$  on the vector  $\mathbf{s}$ . We show that a Nash-equilibrium exists under relatively mild assumptions about the allocation scheme. Specifically, we require that the market share  $\mathbf{a}_i(\mathbf{s})$  of supplier  $i$  is increasing continuous and concave in  $s_i$  and  $\sum_{i=1}^N \mathbf{a}_i(s_i) = 1$ .

An example scheme that satisfies this assumption is one in which market share is allocated proportionally to stocking levels. That is,

$$\mathbf{a}_i(\mathbf{s}) = \frac{s_i}{s_i + \sum_{j \neq i} s_j}. \quad (9)$$

This allocation scheme has the advantage of being based on easily observable quantities and, therefore, easy to implement and to enforce. It also arises naturally in other settings where, for example, demand is stimulated by inventory – see Wang and Gerchack (2001). This scheme is not however informative of the capabilities of the suppliers and, in particular, their abilities to guarantee a

certain level of service. An alternative to the stock-proportional scheme is therefore one proportional to service levels, measured for example, by fill rates. In this case,

$$\mathbf{a}_i(\mathbf{s}) = \frac{f_i}{f_i + \sum_{j \neq i} s_j} . \quad (10)$$

Fill rate is the fraction of customer demand that is immediately met from stock. In our setting, by virtue of the PASTA property,  $f_i = \text{Prob}(I_i > 0) = 1 - \mathbf{r}_i^{s_i}$ . It is not difficult to see that under this scheme  $\mathbf{a}_i$  is both continuous, increasing and concave in  $s_i$ . This allocation scheme is in line with commonly used contracts between suppliers and manufacturers. It also captures more of the characteristics of the suppliers since it involves both stocking levels and capacity. See Boyaci and Gallego (2002) for related discussion. An important question for the manufacturer is which of the two schemes is more advantageous (for example, which of the two induces a higher fill rate for the manufacturer?).

## 5 The Nash Equilibrium

The competition for market share among the suppliers defines a strategic game, where the players are the  $N$  suppliers, the pure strategy space is the set of admissible base-stock levels  $s_i$  for each player  $i$ , and the payoff function is the profit function  $\mathbf{p}_i(\mathbf{s})$  for player  $i$  for each profile  $\mathbf{s} = (s_1, s_2, \dots, s_N)$  of strategies. Each player's objective is to maximize their own profit function, given that each player has full knowledge of the structure of the game. A Nash equilibrium for the competition among the suppliers is any point  $(s_1, s_2, \dots, s_N)$  such that none of the suppliers can increase their profit by unilaterally changing their stocking levels. Therefore, to find an equilibrium point, it suffices to solve  $N$  supplier problems (equation 2) simultaneously. To show the existence of a pure-strategy Nash equilibrium, it is sufficient to show that (a) the strategy space for each player forms a nonempty compact convex set and (b) the profit functions  $\mathbf{p}_i$  for each supplier  $i$  is continuous in  $\mathbf{s}$  and concave in  $s_i$  (see theorem 1.2 of (Fudenberg and Tirole 1991)).

**Theorem 1:** *There exists a pure strategy Nash equilibrium for the game defined by the suppliers' competition.*

**Proof:** The profit function  $\mathbf{p}_i$  is continuous in  $\mathbf{s}$  since  $\mathbf{a}_i(\mathbf{s})$  is continuous in  $\mathbf{s}$ . Note that

$$\frac{\partial^2 \mathbf{p}_i}{\partial S_i^2} = \frac{\partial^2 \mathbf{a}_i}{\partial S_i^2} \mathbf{I} r_i - (h_i + b_i) \frac{\mathbf{r}_i^{s_i+1}}{1 - \mathbf{r}_i} (\ln \mathbf{r}_i)^2 \leq 0 .$$

Hence,  $\mathbf{p}_i$  is concave. To show that the strategy space is continuous, convex and compact, we show that the values of stocking levels from which each supplier chooses are restricted to the range  $[0, s_i^u]$ , where  $s_i^u < 8$ . To see this, we note that the profit of supplier  $i$  when they hold no inventory is given by  $\mathbf{p}_i^0(s_{-i}) = \mathbf{a}_i^0(s_{-i}) \mathbf{I} r_i$  where  $\mathbf{a}_i^0(s_{-i}) = \mathbf{a}_i(s_i = 0, s_{-i})$ , where the notation  $s_{-i}$  is used to denote the strategy profile of the other suppliers. To show that the strategy space is bounded above by  $s_i^u$ , it is sufficient to show that if  $s_i > s_i^u$ , then  $\mathbf{p}_i(s_i, s_{-i}) \leq \mathbf{a}_i^0(s_{-i}) \mathbf{I} r_i$ . If we choose  $s_i^u$  to be the solution of  $\mathbf{I} r_i + g_i - h_i E I_i(s_i^u) - b_i E B_i(s_i^u) = 0$ , then  $\mathbf{p}_i(s_i^u, s_{-i}) \leq \mathbf{I} r_i - h_i E I_i(s_i^u) - b_i E B_i(s_i^u) + g_i \leq \mathbf{p}_i^0(s_{-i})$ . Noting that the equation  $\mathbf{I} r_i + g_i - h_i E I_i(s_i^u) - b_i E B_i(s_i^u) = 0$  admits a unique solution completes the proof.

Showing uniqueness of the equilibrium point without additional assumptions about the allocation scheme is difficult. However, we can show that given a strategy profile  $s_{-i}$  by the other suppliers, supplier  $i$  has a unique optimum stocking level  $s_i^*$ .

**Proposition 1:** For fixed  $s_{-i}$ , there exists a unique optimum stocking level,  $s_i^*$ , that maximizes the profit of player  $i$  which satisfies the condition

$$s_i^* = \frac{\ln[\mathbf{b}_i \frac{h_i - \mathbf{q}_i(s_i^*)}{h_i + b_i}]}{\ln(\mathbf{r}_i)} \text{ for } h_i - \mathbf{q}_i(0) < \frac{h_i + b_i}{\mathbf{b}_i}, \text{ and } s_i^* = 0, \text{ otherwise, and } \mathbf{q}_i(s_i^*) = \mathbf{1} r_i \left. \frac{\partial \mathbf{a}_i(s_i, s_{-i})}{\partial s_i} \right|_{s_i=s_i^*}.$$

The proof is omitted for brevity and can be found in (Benjaafar et al. 2003). An important corollary of this result, and of the fact  $\mathbf{q}_i(s_i^*) > 0$ , is that  $s_i^* > s_i^n$  when  $s_i^* > 0$ , and  $s_i^* = s_i^n = 0$  otherwise.

**Corollary 1:** Competition leads to higher base stock levels at each supplier and consequently to higher fill rates.

If we restrict ourselves to either the stock- or fill rate-proportional allocation schemes described in section 2 and if we limit ourselves to identical suppliers (i.e.,  $h_i = h$ ,  $b_i = b$ ,  $r_i = r$ , and  $\mathbf{r}_i = \mathbf{r}$  for  $i=1, 2, \dots, N$ ) suppliers, then we can show that the Nash equilibrium is unique. The result is stated in the following theorem and the proof can be found in (Benjaafar et al. 2003).

**Theorem 2:** In a system with  $N$  identical suppliers, under either a stock- or fill rate proportional allocation scheme In a system with  $N$  identical suppliers, there exists a unique Nash equilibrium, characterized by base-stock level  $s_i^* = s^*$  at each supplier that satisfies the following equation

$$s^* = \ln[\mathbf{b} \frac{h - \mathbf{q}(s^*)}{h + b}] / \ln(\mathbf{r}).$$

## 5 Stock-Proportional versus Fill Rate-Proportional Market Allocation

In a system with identical suppliers, we show that an allocation proportional to base-stock levels leads to higher fill rates than an allocation proportional to fill rates.

**Theorem 3:** Let  $f_s$  and  $f_f$  denote the Nash equilibrium fill rate under the stock proportional and fill-rate proportional allocation schemes respectively, in a system with  $N$  identical suppliers, then  $f_s = f_f$ .

**Proof:** The Nash equilibrium solution under the stock-proportional and fill rate-proportional allocations satisfy respectively the following equations:

$$f_s = 1 - \frac{\mathbf{b}}{h + b} \left\{ h - \mathbf{1} r \frac{(N-1) \ln(1/\mathbf{r})}{N^2 \ln(1/(1-f_s))} \right\}, \quad (11)$$

and

$$f_f = 1 - \frac{\mathbf{b}}{h + b} \left\{ h - \mathbf{1} r \frac{(N-1) \ln(1/\mathbf{r})}{N^2} \frac{(1-f_f)}{f_f} \right\}. \quad (12)$$

Using the fact that for  $x < 1$ , we have  $\ln[1/(1-x)] < x/(1-x)$ , it is not difficult to show that  $f_s > f_f$ .

An intuitive explanation for this result is that a stock-proportional allocation is more sensitive to changes in stocking levels than one proportional to fill-rates (the rate of increase of  $\mathbf{a}_i(s_i, s_{-i})$  in  $s_i$  for fixed  $s_{-i}$  is higher under a stock-proportional allocation). For manufacturers, this means that there is value in constructing the terms of the competition to reward higher stocking levels than higher fill rates.

## 6 The effect of Number of Suppliers

For  $N$  identical suppliers ( $N = 2$ ) and a stock-proportional market share allocation, the equilibrium base-stock level solves the following equation

$$s^* = \ln\left[\mathbf{b} \frac{h - \mathbf{q}(s^*)}{h + b}\right] / \ln(\mathbf{r}), \quad (13)$$

where in this case  $\mathbf{q}^*(s^*) = \mathbf{I}r \frac{N-1}{N^2 s^*}$ , from which we can see that as  $N$  increases  $s^*$  decreases. In the limit case, we have

$$\lim_{N \rightarrow \infty} s^* = \ln\left[\mathbf{b} \frac{h}{h + b}\right] / \ln(\mathbf{r}). \quad (14)$$

Note that this limit equilibrium base stock is the base stock level obtained in section 3 for the case of no competition. A similar result can be obtained for the fill-rate proportional allocation.

**Theorem 4:** *In a system with  $N$  identical suppliers, the equilibrium stocking level  $s^*$  (and therefore the equilibrium fill rate) are decreasing in  $N$ . As  $N \rightarrow \infty$ ,  $s^* \rightarrow s^n$ , the optimal stocking level without competition.*

Noting that when  $N = 1$ ,  $s^* = s^n$ , the number of suppliers that maximizes the fill rate is 2. This means that for manufacturers, the optimal number of suppliers is 2 (a duopoly).

## 6. Conclusion

In this paper, we present models for competition among multiple suppliers for market share from a single manufacturer. The suppliers produce to stock a single product and are allocated demand by the manufacturer based on the amount of inventory they hold. We prove the existence of a Nash equilibrium for a broad class of market allocation schemes. For the special case of identical suppliers and either stock-proportional or fill rate-proportional allocations, we show the uniqueness of the Nash equilibrium. Analysis of the Nash equilibrium for these cases reveals that (a) the manufacturer benefits from competition (in the form of higher fill rates), (b) the manufacturer benefits more from a stock-proportional allocation than a fill rate-proportional allocation, and (c) the manufacturer benefits the most when the number of suppliers is two.

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